Econ 306
Professor Coughlin

Homework 9

Directions:

I. Answer questions 1-3 (1 point each)
II. Please complete the homework 9 problems on the Aplia Website. (2 Points)
III. Complete 29.1 (3 pts), 29.3 (2 pts) below.

1. A game has two players. Each player has two possible strategies. One strategy is called "cooperate", the other is called "defect". Each player writes on a piece of paper either a C for cooperate or a D for defect. If both players write C, they both get a payoff of $100. If both players defect they each get a payoff of $0. If one player cooperates and the other player defects, the cooperating player gets a payoff of S and the defecting player gets a payoff of T. To defect, D, will be a dominant strategy for both players if:
a) S+T > $100
b) T > 2S
c) S < $0 and T > $100
d) S > T and T > $100
e) S and T are any positive numbers

Use the following information for questions 2 and 3.

Consider the following zero-sum game. The coach of the offensive football team has two options on the next play. He can run the ball or he can pass. His rival can defend either against the run or against the pass. Suppose that the offense passes. Then if the defense defends against the pass, the offense will make zero yards, and if the defense defends against the run, the offense will make 25 yards. Suppose that the offense chooses to run. If the defense defends against the pass, the offense will make 10 yards, and if the defense defends against a run, the offense will gain 2 yards.

2. Write down a payoff matrix for this game.

3. Is there a Nash equilibrium in pure strategies for this game? Is there a mixed strategy Nash equilibrium for this game? Find all Nash equilibria.

Calculus **29.1 (2)** Two software companies sell competing products. These products are substitutes, so that the number of units that either company sells is a decreasing function of its own price and an increasing function of the other product's price. Let p_1 be the price and x_1 the quantity sold of product 1 and let p_2 and x_2 be the price and quantity sold of product 2. Then $x_1 = 1000\left(90 - \frac{1}{2}p_1 + \frac{1}{4}p_2\right)$ and $x_2 = 1000\left(90 - \frac{1}{2}p_2 + \frac{1}{4}p_1\right)$. Each company has incurred a fixed cost for designing their software and writing the programs, but the cost of selling to an extra user is zero. Therefore each company will maximize its profits by choosing the price that maximizes its total revenue.

(a) Write an expression for the total revenue of company 1, as a function of the its price p_1 and the other company's price p_2._____.

(b) Company 1's *best response function* $BR_1(\cdot)$ is defined so that $BR_1(p_2)$ is the price for product 1 that maximizes company 1's revenue given that the price of product 2 is p_2. With the revenue functions we have specified, the best response function of company 1 is described by the formula $BR_1(p_2) =$_____. (Hint: Take a derivative of revenue with respect to p_1 and solve for the revenue-maximizing p_1 given p_2.)

(c) Use a similar method to solve for company 2's best response function $BR_2(p_1) =$_____.

(d) Solve for the Nash equilibrium prices $p_1 =$_____ and $p_2 =$_____.

(e) Suppose that company 1 sets its price first. Company 2 knows the price p_1 that company 1 has chosen and it knows that company 1 will not change this price. If company 2 sets its price so as to maximize its revenue given that company 1's price is p_1, then what price will company 2 choose? $p_2 =$_____ If company 1 is aware of how company 2 will react to its own choice of price, what price will company 1 choose? _____ Given this price for company 1, what price will company 2 choose?_____.

29.3 (1) This is an example of the game of "Chicken." Two teenagers in souped-up cars drive toward each other at great speed. The first one to swerve out of the road is "chicken." The best thing that can happen to you is that the other guy swerves and you don't. Then you are the hero and the other guy is the chicken. If you both swerve, you are both chickens. If neither swerves, you both end up in the hospital. A payoff matrix for a chicken-type game is the following.

Chicken

		Leroy	
		Swerve	Don't Swerve
Joe Bob	Swerve	1, 1	1, 2
	Don't Swerve	2, 1	0, 0

(a) Does this game have a dominant strategy?_____ What are the two Nash equilibria in pure strategies?_____.

(b) Find a Nash equilibrium in mixed strategies for this game._____.