Economics 662: Theoretical IO
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1 Lecture 1: September 1, 2005

1.1 Course Outline

- The course is laid out according to market structure: from monopoly to oligopoly to perfect competition.
- Monopoly
  - (1) Price discrimination.
  - (2) Product choice.
  - (3) Bundling.
- Oligopoly
  - (1) Classic oligopoly theory (Cournot and Bertrand).
  - (2) Effects of horizontal mergers (antitrust).
  - (3) Dynamic oligopoly (bundling as a deterrent strategy).
  - (4) Models of predation.
  - (5) Repeated oligopoly (implicit or explicit collusion).
- Perfectly competitive markets. We consider the size and distribution of firms in competitive markets.
- Additions (if time permits) include vertical restraints, R and D, and network effects.
- So how does IO fit into the classical (General Equilibrium) model of consumer and producer behavior? While trade theory takes GE theory and simply applies it to trade topics, IO theory relaxes some of the assumptions of GE:
  - (1) What if firms do not maximize profits due to some agency problem ?
  - (2) Distortions may arise due to informational barriers.
  - (3) Distortions may arise due to non-convexities.
  - (4) Distortions may arise due to firms having power over price.
  - (5) Distortions may arise due to externalities.
  - (6) Distortions may arise due to legal conditions that affect behavior.

  There is no good unifying theory of IO so we just study a set of particular cases.

1.2 Part I: Monopoly

- Why do monopolies exist? Usually due to some sort of economies of scale where it is most efficient to have a single supplier of a good.

- We say a cost function is sub-additive if for all $q = (q_1, \ldots, q_n)$, we have:

$$\sum_{i=1}^{n} C(q_i) > C(\sum_{i=1}^{n} q_i).$$

- Consider a social planner choosing q to :

$$Max\ \{u(\sum q_i) - \sum C(q_i)\}.$$

  If $C(\cdot)$ is sub-additive, then the SP will set $q_i > 0$ and $q_{-i} = 0$ for some i. We might get this result if there were fixed costs and constant marginal costs.

- However it is not obvious if the SP solution is privately and socially efficient due to private costs being born by the monopolist.

- Examples of monopolies in the US:
  - (1) Utility companies. Huge economies of scale in owning the infrastructure. The source of the monopoly is in the distribution of the service, but we could still sell the right to provide the electricity for example.
  - (2) Coke/Pepsi choice on a college campus. UMD receives money from pepsi and pepsi is given some monopoly power over the pop distribution on campus. Everyone wins except the students!
  - (3) Patents - intellectual or otherwise. This is a legally sanctioned monopoly. There are huge fixed costs to production with very small (zero) marginal costs. Clearly, a sub-additive cost function.

- So what are the key characteristics of a monopoly?
  - The firm must have market power so his decision affects the market.
  - The firm must also NOT incorporate the strategic response of other firms into his decision. This is like a myopic oligopolist.

- So consider a monopolist facing inverse demand curve, $P(Q)$ with cost curve $C(Q)$. The problem of the monopolist is:

$$Max_Q\ \{P(Q)Q - C(Q)\}.$$

  If the objective is 1) continuous, 2) differentiable, and 3) quasi-concave, then we have the FOC:

$$QP'(Q) + P(Q) - C'(Q) = 0.$$

- We could also write this problem:

$$Max_P \, \{PQ(P) - C(Q(P))\},$$

with $P = P(Q(P))$, which exists if $P' < 0$.

- So note the FOC is just the usual result of $MR = MC$. In the case of PCM, $P'(Q) = 0$ and we get $P = MC$. Rearrange the FOC:

$$P - C'(Q) = -QP'(Q) = -Q\frac{dP}{dQ}.$$

Or,

$$\frac{P - C'(Q)}{P} = -\frac{Q}{P}\frac{dP}{dQ} = -\frac{1}{P/Q * dQ/dP}.$$

So,

$$\underbrace{\frac{P - C'(Q)}{P}}_{Markup} = \underbrace{-\frac{1}{\epsilon}}_{1 \text{ over elasticity}}.$$

And this is a necessary condition for profit maximization. As $\epsilon \to \infty$, as with a PCM, the markup goes to zero.

- More next time.

2 Lecture 2: September 6, 2005

2.1 More on Monopoly Pricing

Learner Index

- Recall the FOC of the monopolist:

$$\frac{P - C'}{P} = -\frac{1}{\epsilon}.$$

This implies a monopolist will never produce on an inelastic portion of the demand curve. Since the LHS is less than 1, this means ϵ is greater than one.

Comparative Statics on Costs

- Consider a monopolist who has one of two cost schedules, C_1 and C_2 with $C_2' \geq C_1'$. Maybe he may or may not face a tax by the government. Let q_1 and q_2 be the respective profit maximizing selections (may or may not be unique) under these two costs.
- Intuitively, we might think that $q_2 \leq q_1$ as the monopolist cuts back production when costs are higher. However, to guarantee this, we would need some sort of convexity assumption on the demand curve or cost curve.
- However, we can get at this inequality another way. Let $p_1 = p(q_1)$ and $p_2 = p(q_2)$ be the prices at the optimal quantities. Then, by optimization:

$$p_1 q_1 - c_1(q_1) \geq p_2 q_2 - c_1(q_2),$$

because profits at (p_1, q_1) must be larger than any other price/quantity combo including (p_2, q_2). Similarly:

$$p_1 q_1 - c_2(q_1) \leq p_2 q_2 - c_2(q_2).$$

Subtracting these equations, we get:

$$c_2(q_1) - c_1(q_1) \geq c_2(q_2) - c_1(q_2).$$

Or,

$$c_2(q_1) - c_2(q_2) - (c_1(q_1) - c_1(q_2)) \geq 0.$$

Which can be written:

$$\int_{q_2}^{q_1} c_2'(x)dx - \int_{q_2}^{q_1} c_1'(x)dx \geq 0.$$

$$\int_{q_2}^{q_1} \underbrace{[c_2'(x) - c_1'(x)]}_{\geq 0 \text{ by ass.}} dx \geq 0.$$

Hence if $q_2 > q_1$, the area would have to be negative which would violate this last condition. Thus $q_2 \leq q_1$ as required.

- We don't require and convexity assumptions to get this intuitive result.

Multi-product Monopoly

- It's very common for a monopolist to sell many products. The choice variable is now $p \in \Re^n_+$ and assume costs are $C : \Re^n_+ \mapsto \Re_+$. There are n different demand curves so $D_i : \Re^n_+ \mapsto \Re_+$ for $i = 1 \dots n$.

- The monopolist's problem is thus:

$$Max_{p \in \Re^n_+} \{\sum_{i=1}^{n} p_i D_i(p) - C(D_1(p), \dots, D_n(p))\}.$$

  Assume $q_i = D_i(p)$ so the monopolist sells all its production.

- FOC for p_i (interior solution):

$$D_i(p) + p_i \frac{\partial D_i(p)}{\partial p_i} + \sum_{j \neq i} p_j \frac{\partial D_j(p)}{\partial p_i} - \sum_{i=1}^{n} \frac{\partial C(q)}{\partial q_j} \frac{\partial D_j(p)}{\partial p_i} = 0.$$

  Pulling out the i^{th} term out of the cost function's derivative, the FOC becomes:

$$0 = \underbrace{D_i(p) + p_i \frac{\partial D_i(p)}{\partial p_i} - \frac{\partial C(q)}{\partial q_i} \frac{\partial D_i(p)}{\partial p_i}}_{FOC\ from\ market\ i} + \sum_{j \neq i} (p_j - \frac{\partial C(q)}{\partial q_j}) \frac{\partial D_j(p)}{\partial p_i}.$$

  So the first term is the FOC from the single product monopolist but we also have these other terms.

- Suppose $D_i(p) = D_i(p_i)$, so the demand for product i is only a function of its own price. Then the FOC of the multi-product monopolist will reduce to a sequence of FOCs, all looking like the FOC for a single product monopolist. Here we still MAY HAVE COST SYNERGIES! Markets are related in production but not in demand. There is something going on here which he may address next time. An example of cost synergies might be software production: producing both word and excel are less costly than two firms producing them separately.

- Now suppose $C(q) = \sum_i C_i(q_i)$ but $D_i(p)$ is still a function of all prices. Now there are NO cost synergies but there are demand synergies.

- Example of demand synergies: consider a multi-product monopolist who produces two goods with costs, $c_1(q_1) = cq_1$ and $c_2(q_2) = cq_2$ and demand:

$$q_1(p_1, p_2) = a + bp_2 - dp_1.$$

$$q_2(p_1, p_2) = A + Bp_1 - Dp_2.$$

Assume $d, D > 0$. Thus profits are:

$$\pi(p_1, p_2) = (p_1 - c)(a + bp_2 - dp_1) + (p_2 - c)(A + Bp_1 - Dp_2).$$

FOC(p_1):

$$a + bp_2 - dp_1 - dp_1 + dc + Bp_2 - cB = 0.$$

$$a + bp_2 + dc - 2dp_1 + B(p_2 - c) = 0.$$

So if there are no synergies, $B = 0$, and we get something like G-2.1, setting $MR_1 - MC_1 = 0$, we get $\hat{p}_1$. However when $B > 0$, we have substitutes and the solution looks more like G-2.2. Since $B(p_2 - c)$ is positive, that means on the line in G-2.1, we must be at a negative point on that line. Hence $\hat{p}_{1,B>0}$ shifts to the right (and quantity produced of good one must fall). Hence we should see higher prices if the monopolist produces substitute goods and lower prices if he produces complements (compared to the case of NO demand synergies).

Costs of a Monopoly

- One possible measure of the social costs of a monopoly is the dead-weight loss (DWL) as illustrated in G-2.3. Harberger estimated the DWL to be very small in most monopolies.

- What about the learner index of 1 over the elasticity of demand? Consider G-2.4. Here we have a HIGHLY inelastic demand curve so ϵ is very small which makes the markup very large. However, the monopolist will probably produce at something like the point E in the graph and the competitive quantity is almost exactly equal to the monopolist's quantity. So while the monopolist has a lot of power over price, he still chooses something very close to the competitive equilibrium. The same is true for a very elastic demand curve like G-2.5. Hence the learner index is not a very good measure of the costs of monopoly.

- There is also the Posner view of the costs of monopoly: why is the firm a monopolist in the first place? He assumes that there is some cost to becoming a monopolist (bribing congressmen, extensive R and D, etc) and all monopoly profits should also be thought of as the cost of attaining this position of market power. Thus the loss to society is both the DWL and this other cost the monopolist bares.

3 Lecture 3: September 8, 2005

3.1 More on Monopoly

Double Marginalization in Monopoly

- If both an upstream and downstream firm are monopolists, the price that the consumer ultimately pays may be subject to multiple monopoly markups.
- Example: A retailer faces demand: $P = A - BQ$ and faces an input cost of w per unit of production. Thus, the firm's problem is:

$$Max_Q \ \{(A - BQ)Q - wQ\}.$$

  FOC:

$$A - 2BQ - w = 0 \Longrightarrow Q^* = \frac{A - w}{2B}.$$

- See G-3.1. Note the perfectly competitive quantity would be $Q^c = \dfrac{A - w}{B}$ which results from $P = MC = w$.
- We can think of Q^* being the derived demand for the upstream firm. Thus the upstream's problem is:

$$Max_w \ \{w(\frac{A - w}{2B})\},$$

  which induces:

$$w^* = \frac{A}{2}.$$

  (Note the upstream firm faces no costs.)

- Hence the downstream quantity becomes:

$$Q^*(w^*) = \frac{A - w^*}{2B} = \frac{A}{4B}.$$

- What about a social planner setting optimal quantity. For the upstream firm who faces no costs, $P = MC$ implies:

$$Q^{sp} = \frac{A}{B}.$$

- So see G-3.2 for a picture of the double marginalization problem. If markets are vertically related which induces this double marginalization structure, the costs of a monopoly may be significant (unlike Harbinger's view).
- **Remarks** Note it is in the interest of the downstream firm to charge a single price to all consumers because differentiating consumers is impossible. However, the upstream firm charging a per unit price may NOT be optimal - a 2 part tariff discussed next will be a better strategy. We need an expansion of the contracting space to reach this solution.

- What about bargaining power between the up and downstream firms? So far we have had the upstream firm making a Take It or Leave It (TILI) offer which means we have given that firm a lot of strategic power. We might want to introduce bargaining into the problem where presumable the optimal quantity would be the monopoly quantity (maximizing total surplus) and the bargaining process would determine the split of the profits going to each firm. But this makes things pretty complicated.

3.2 Price Discrimination

- Consider G-3.3 of a ordinary monopolist charging a uniform price. There is still consumer surplus in this market going to the really high value customers (monopolists face a strategic constraint in not being able to charge different prices to different consumers).

- In G-3.4 we have the full extraction price discrimination case where the demand curve is now equal to the marginal revenue curve. The monopolist now extracts ALL consumer surplus (CS).

- Price Discrimination (PD) is difficult to place a precise definition on, but it may be defined as charging a different price to each consumer for the same product and the difference in price is not based on cost. Another definition might be: "You know it when you see it!"

- What are the necessary conditions for PD? There are 3:
  - (1) Need market power ($P > MC$).
  - (2) Some mechanism exists to identify and sort consumers.
  - (3) No arbitrage is possible.

- Pigou classifies PD into three categories:
  - (1) 1^{st} degree PD: full rent (CS) extraction.
  - (2) 2^{nd} degree PD: consumers sort endogenously (eg, sell at low price for high quantity consumer).
  - (3) 3^{rd} degree PD: consumers sort exogenously (eg, sell at different prices to men and women).

- A classic example of first degree price discrimination (FDPD) is as follows. A monopolist produces output x at cost $c(x)$ and all units are measured in units of a numeraire good, y (could be money). Initially a consumer has no units of x and $\overline{y}$ units of y, so his reservation utility is $u(x, y) = u(0, \overline{y})$. A monopolist makes a TILI offer (x, p) which the consumer accepts if:

$$u(x, \overline{y} - p) \geq u(0, \overline{y}). \quad (1)$$

- This is called a "Forcing Contract" since the monopolist has a strong strategic power. The monopolist also has an important informational power: he knowns $u(\cdot)$ and $\overline{y}$.

- Thus the monopolist's problem is:

$$Max_{(p,x)}\ \{p - c(x)\},\ s.t.\ (1).$$

Our lagrangian:

$$\mathcal{L} = p - c(x) + \lambda[u(x, \overline{y} - p) - u(0, \overline{y})].$$

FOC(x):

$$\lambda^* \frac{\partial u(x^*, \overline{y} - p^*)}{\partial x} = c'(x^*).$$

FOC(p):

$$\lambda^* \frac{\partial u(x^*, \overline{y} - p^*)}{\partial p} = 1.$$

- If $u(\cdot, \cdot)$ is strictly increasing in both args, then $\lambda^* > 0$ and the IR constraint must bind. Thus:

$$u(x^*, \overline{y} - p^*) = u(0, \overline{y}).$$

Also, dividing the FOCs:

$$\frac{u_1(x^*, \overline{y} - p^*)}{u_2(x^*, \overline{y} - p^*)} = \frac{c'(x^*)}{1},$$

Or the MRS between x and y equals the MRTS between x and y, our usual optimality condition.

- So FDPD eliminates the distortion of the monopolist since he sells up until the competitive quantity. This is a first welfare theorem result that the solution is efficient but in terms of fairness of the allocation, the story is quite different (a second welfare theorem story).

- Now suppose the utility function is quasilinear in y, money. So:

$$u(x, y) = v(x) + y,\ v' > 0,\ v'' < 0.$$

We now weaken the monopoly power of the firm from a forcing contract to a two part tariff. The firm offers a fixed entry fee and per unit price combination of (A, p). This is like the costco or sprint situation. We formulate the problem such that if a consumer buys a positive amount of x from the monopolist, he pays $A + px$.

- A consumer who pays A up front and chooses x, solves:

$$Max_x\ \{v(x) + \overline{y} - A - px\},$$

which induces:

$$p = v'(x^*).$$

Note that $v'(x^*)$ is the consumer's inverse demand function for the good x.

- Given the consumer's reservation utility, $u(0, \overline{y}) = v(0) + \overline{y}$, his net gain from buying is:
$$v(x^*) + \overline{y} - A - px^* - (v(0) + \overline{y}) = v(x^*) - v(0) - px^* - A.$$
Or,
$$\int_0^{x^*} v'(s)ds - px^* - A = \phi.$$
The integral term is literally the consumer surplus for the consumer from buying the good. For buying x to be optimal, it must be that $\phi \geq 0$.

- Since the monopolist can charge the 2 part tariff to each consumer separatly, he can still extract all consumer surplus and achieve the first best solution. The monopolist will simply set $p^* = c'(x^*)$ where x^* maximizes the total surplus. Thus p^* will be constant across all consumers but A will vary for each.

- What if we need to charge the same (A, p) combination to ALL consumers? Maybe because we cannot distinguish between consumers or discrimination is illegal? Consider the Disneyland Dilemma. They offer a two part tariff (A, p) but $p = 0$! So once you enter the park, you don't pay to go on each and every ride. The marginal cost of running each ride must be greater than zero so why does disney not charge for each ride (besides the inconvenience of it) ?

- Suppose the market consists of equal amounts of two types of consumers: high and low demand. See G-3.5. It would appear that if disney couldn't discriminate, he would have to set A equal to the CS surplus region of the low type. Profits would then equal $2A$ (for the high and low types). The author goes on to say that setting a price a bit above marginal cost seems to be optimal because of the increased profits, but for crazy representations of the demand curves, it may actually be optimal to set the price BELOW marginal cost (maybe zero) and just charge a fixed entry fee. I'm not convinced.

4 Lecture 4: September 13, 2005

4.1 More on Price Discrimination

3^{rd} Degree Price Discrimination (TDPD)

- Here we have linear though non-uniform pricing. The monopolist can distinguish between consumer segments but he cannot offer a complicated contract (like a 2 part tariff).
- There are therefore no demand synergies but there may be cost synergies.
- Consider a model with $i = 1 \dots m$ markets. Optimality in market i implies:

$$\frac{p_i^* - c'(q)}{p_i^*} = -\frac{1}{\epsilon_i}.$$

- Two examples might be highly geographically separated markets, or a product which uses different voltages in different countries so separation is natural.
- See G-4.1 for two markets with different prices and then a uniform price if this was mandated. Note that since uniform pricing is an option to a monopolist who practices TDPD, due to revealed preference, the monopolist must be strictly worse off under a mandated uniform price.
- One question we might ask is what price will a monopolist choose if uniform pricing is mandated? Will it be some intermediate price between the min and max of his prices before the policy? With two market segments, this seems intuitive, but with 3 or more segments, we need a specific assumption to get an intermediate uniform price. See G-4.2. If individual profit functions are all quasi-concave, then all of the profit functions to the LEFT of p_{min} must be increasing and all profit functions to the right of p_{max} must be decreasing. Hence, a mandated uniform price will have to lie in the interior of $\{p_{min}, p_{max}\}$.
- What about the welfare consequences of imposing a uniform price on an otherwise third degree price discriminating monopolist? Turns out it is hard to quantify, but we do have the following result.

  A necessary condition for TDPD to be welfare improving over uniform pricing

- The punchline as we will see is that if total quantity produced rises under TDPD, then the uniform pricing mandate would actually lead to lower overall welfare.
- Consider a monopolist with constant marginal cost, c so:

$$C(\sum_i q_i) = c \sum_i q_i.$$

The firm faces n separate markets with $D_i(p) = D_i(p_i)$ and $D'(p_i) \leq 0$. There are two possible regimes:

TDPD Regime: yields (p_i, q_i) in each market with $q_i = D_i(p_i)$.

Uniform Regime: yields $(\overline{p}, \overline{q}_i)$ in each market with $\overline{q}_i = D_i(\overline{p})$.

- **Aside on Consumer Surplus** Here we show that CS in each market is convex in prices. Let $CS_i(p_i)$ denote the CS in market i. See G-4.3. Thus,

$$CS_i(p_i) = \int_{p_i}^{\infty} D_i(x)dx.$$

So:

$$CS_i'(p_i) = -D_i(p_i),$$

and,

$$CS_i''(p_i) = -D_i'(p_i) \geq 0.$$

Thus CS is convex! An equivalent definition of a convex function is for the subgradient of the function to lie below the function itself. See G-4.4. Here, by definition:

$$f'(p) = \frac{f(p) - K}{\overline{p} - p},$$

so:

$$f'(p)(\overline{p} - p) = f(p) - K.$$

Then:

$$f'(p)(\overline{p} - p) \geq f(p) - f(\overline{p}),$$

because $K < f(\overline{p})$. We'll use this result in the next section.

- Now consider the change in welfare from TDPD compared with uniform pricing:

$$\Delta W = \sum_{i=1}^{n} CS_i(p_i) - CS(\overline{p}) + \sum_{i=1}^{n} (p_i - c)q_i - (\overline{p} - c)\overline{q}_i.$$

So if this is positive, then TDPD increases total welfare. Since CS is convex, applying our subgradient result:

$$CS_i(p_i) - CS(\overline{p}) \geq CS_i'(\overline{p})(p_i - \overline{p}).$$

Thus,

$$\Delta W \geq \sum_{i=1}^{n} CS_i'(\overline{p})(p_i - \overline{p}) + p_i q_i - \overline{pq}_i - c(q_i - \overline{q}_i).$$

Now let $\Delta q_i = q_i - \overline{q}_i$ and note that $CS_i'(\overline{p}) = -D_i(\overline{p}) = -\overline{q}_i$. We have:

$$\Delta W \geq \sum_{i=1}^{n}[-\overline{q}_i(p_i - \overline{p}) + p_i q_i - \overline{p}\overline{q}_i - c\Delta q_i].$$

$$\Delta W \geq \sum_{i=1}^{n}[-\overline{q}_i p_i + p_i q_i - c\Delta q_i].$$

$$\Delta W \geq \sum_{i=1}^{n}[p_i(q_i - \overline{q}_i) - c\Delta q_i].$$

$$\Delta W \geq \sum_{i=1}^{n}[p_i(q_i - \overline{q}_i) - c\Delta q_i].$$

$$\Delta W \geq \sum_{i=1}^{n}(p_i - c)\Delta q_i.$$

- Similarly, applying the subgradient method in reverse yields:

$$\Delta W \leq \sum_{i=1}^{n}(\overline{p} - c)\Delta q_i.$$

- Bringing these two inequalities together yields:

$$\sum_{i=1}^{n}(p_i - c)\Delta q_i \leq \Delta W \leq \sum_{i=1}^{n}\underbrace{(\overline{p} - c)}_{positive}\Delta q_i.$$

The second inequality represents an upper bound on the change in welfare. For TDPD to be welfare improving, we need ΔW to be positive. Therefore we need $\sum_i \Delta q_i$ to be positive for TDPD to even have a chance of being welfare improving. Again this is just a necessary condition, and though we can't say anything definitive about the consequences of TDPD, it looks like the bias in terms of welfare, is towards uniform pricing.

- So what is the view of TDPD in terms of US policy and law? TDPD is always ok in final goods market, but for intermediate goods, there is an act that forbids discrimination in some cases.

- The Robinson Patman Act restricts TDPD when the following 5 conditions are met:
  - (1) Must be an intermediate goods market.
  - (2) There must be interstate trade of the good for the feds to step in.
  - (3) The goods in question must be of like quality.
  - (4) There must be a significant difference in prices which is cost independent.

  - (5) There must be some sort of "injury to competition."

- A firm found guilty under this act is subject to treble damages so it's fairly draconian punishment.

- Consider the chainstore setup as in G-4.5. We have wholesalers (A, B, and C), retailers (a, b, c, d) and consumers. a might be walmart and b might be a Ma and Pa store competing with the retail giant. A primary injury would involve TDPD from A to his customer, say a. Secondary is when A price discriminates with his customer a, but has damaging effects on b, and finally tertiary injuries are those that hit the consumers. The Ma and Pa stores tried to invoke Robinson Patman against the retail giants and were successful to some extent.

- The problem with enforcing this act is distinguishing TDPD from just good competitive pricing practices. It's hard to prove to a jury that you're lowering your price simply to compete in the market. One famous case is the Utah Pie (UP) Case where UP was selling premade pies in the Salt Lake area and had a pretty strong monopoly on the area. A competitor from California, Continential Baking (CB) decided to enter the market in Salt Lake. CB undercut UP in the Salt Lake market while their California prices remained relatively higher. UP sued CB under the above act and won significant damages. Was CB just competing with UP or were they price discriminating? This is an anti-trust law coming out that allows a monopolist to retain it's monopoly power. Seems counter-intuitive. In reality, only a very small number of cases have ever been successfully decided using the Robinson Patman Act.

5 Lecture 5: September 5, 2005

5.1 More on Price Discrimination

More 3^{rd} Degree Price Discrimination (TDPD)

- Moving from TDPD to uniform pricing tends to lower high prices and raise low prices. Uniform pricing has the better intuitive welfare feature that at equilibrium, the marginal rates of substitution are the same for all segments of the market. This is not the case with TDPD.

- Example where TDPD pareto dominates uniform pricing. See G-5.1. We have two markets with $P_h = 4 - Q$ and $P_l = 1 - Q$ and the monopolist has no costs. The total mass of high and low type consumers is M_h and M_l. Clearly an ordinary monopolist would set $p_h^* = 2$ and $p_l^* = \frac{1}{2}$ if he could third degree price discriminate. Under uniform pricing, the monopolist solves:

$$Max_p \{M_h * p(4-p) + M_l * p * max\{0, 1-p\}\},$$

since it is possible the monopolist will choose not to sell to low types. The FOC if he sells to both markets is:

$$M_h(4-2p) + M_l(1-2p) = 0 \Longrightarrow \hat{p} = \frac{4M_h + Ml}{2(M_h + M_l)} = \frac{4(M_h/M_l) + 1}{2(M_h/M_l) + 2}.$$

So as $M_l \to 0, \hat{p} \to 2$. As $M_h \to 0$, $\hat{p} \to \frac{1}{2}$. But notice that the monopolist would NEVER choose a price of 1. At that price, the low demanders are priced out of the market, but the price is set below the price he charged the high type under TDPD. So if $M_h >> M_l$, the optimal price is $\hat{p}^* = 2$. What can we say about consumer welfare? Low types are clearly worse off under uniform pricing, high types are the same, and the monopolist is strictly worse off. Thus TDPD pareto dominates uniform pricing.

- Consider another argument (due to Katz) of an upstream monopolist selling to two downstream firms at prices w_1 and w_2: one a local independent grocery store (firm 2), and a large chain store (firm 1). The downstream firms are cournot competitors. The profit functions of the two downstream firms are:

$$\pi_1(w_1, w_2),\ and\ \pi_2(w_1, w_2).$$

Note each firm's profits depend on the price offered to the other firm. Firm 1 is assumed to have the option of vertically integrating into the market of the upstream firm. It faces a large fixed cost of doing so and this prevents firm 2 from doing the same.

- Consider 2 assumptions:

- Assumption 1: Let $u(w_1, w_2)$ be the equilibrium profits of the upstream monopolist when the downstream firms pay w_1 and w_2. $u(\cdot)$ is quasiconcave and has an unconstrained (symmetric) optimum at (w^*, w^*).
- Assumption 2: $\frac{\partial \pi_i}{\partial w_i} < 0$, and $\frac{\partial \pi_i}{\partial w_j} > 0$ for $(i, j) = (1, 2)$ and $(i, j) = (2, 1)$. So each firm's profits are decreasing in their own price and increasing in the other's price.

- See G-5.2. We have isoprofit lines of the upstream firm with a bliss point at (w^*, w^*).
- Let $\Re$ be the equilibrium profits of firm 1 if it vertically integrates. Thus firm 1 will integrate if:

$$\Re > \pi_1(w_1, w_2).$$

- Consider the prices (w_1, w_2) that satisfy:

$$\Re = \pi_1(w_1, w_2).$$

Let $I(w_1)$ be the price that firm 2 would have to get to make firm 1 just indifferent about integrating or not. Thus,

$$\Re = \pi_1(w_1, I(w_1)).$$

Differentiate w.r.t. w_1:

$$0 = \frac{\partial \pi_1}{\partial w_1} + \frac{\partial \pi_1}{\partial w_2}\frac{\partial I}{\partial w_1}.$$

So:

$$\frac{\partial I}{\partial w_1} = I'(w_1) = -\frac{\partial \pi_1/\partial w_1}{\partial \pi_1/\partial w_2} > 0.$$

So $I(w_1)$ is upward sloping.

- See G-5.3. For firm 1 not to integrate, we must be somewhere to the left of the $I(w_1)$ curve. Consider two cases. If uniform pricing is required, then the equilibrium will lie on the 45 degree line so the maximized pricing scheme is at point E. This is well below the bliss point. Without uniform pricing, the upstream firm will choose a point like F, where the $I(w_1)$ curve is tangent to the highest isoprofit line of the upstream firm. Notice that the small grocery store faces a much larger input price $w_2 > w_1$, and both w_1 and w_2 are higher than under uniform pricing case. So TDPD has large negative welfare consequences.

2^{nd} Degree Price Discrimination (SDPD)

- Now we have endogenous price discrimination where the characteristics of the consumers come out of some sort of choice they have to make.
- This is really the result of adverse selection mechanism design problems.

- A monopolist faces a market with two types of consumers indexed by $\theta_H > \theta_L$. Preferences of the consumers are:

$$u(q, T; \theta) = \theta v(q) - T,$$

where q is the quantity of the product purchased, T is the monetary cost of those units, and $v(q)$ is strictly increasing and strictly concave.

- The monopolist has constant marginal cost, $c > 0$. Assume there exists a $q^* > 0$, such that:

$$v'(q^*) = c.$$

- Informational assumption: consumers know their types but the monopolist does not. The monopolist does know:

$$Prob(\theta = \theta_H) = \lambda.$$

- Consider the level sets of the consumer's utility. Totally differentiating:

$$du = 0 \Longrightarrow 0 = \theta v'(q)dq - dT \Longrightarrow \frac{dT}{dq} = \theta v'(q).$$

See G-5.4. In (q, T) space, utility is increasing down and to the right, and the high type's indifference curves are steeper than the low type's at all q. Thus the single crossing property (SCP) is satisfied. Note the slope is INDEPENDENT OF T because the utility is quasilinear in T.

- Now consider the level sets of the monopolist's profits, $\pi(q, T) = T - cq$. Totally differentiating:

$$d\pi = 0 \Longrightarrow 0 = dT - cdq \Longrightarrow \frac{dT}{dq} = c,$$

so the isoprofits lines are straight lines with slope c, increasing up and to the left. See G-5.5.

- At an equilibrium, the level sets will be tangent to the indifference curves if θ is observed. In fact, at the equilibrium,

$$c = \theta_H v'(q_H^*) \Longrightarrow q_H^* = v'^{-1}\left(\frac{c}{\theta_H}\right).$$

$$c = \theta_L v'(q_L^*) \Longrightarrow q_L^* = v'^{-1}\left(\frac{c}{\theta_L}\right).$$

Since $\theta_H > \theta_L$ then $\frac{c}{\theta_H} < \frac{c}{\theta_L}$, so

$$q_H^* = v'^{-1}\left(\frac{c}{\theta_H}\right) > v'^{-1}\left(\frac{c}{\theta_L}\right) = q_L^*.$$

$$q_H^* > q_L^*.$$

6 Lecture 6: September 20, 2005

6.1 More on Price Discrimination

2^{nd} Degree Price Discrimination (SDPD)

- Recall our setup from last time with consumers differing in their preferences with:

$$u(q, t; \theta) = \theta v(q) - T, \;\; v' > 0, \; v'' < 0, \; v(0) = 0.$$

  And $\theta \in \{\theta_L, \theta_H\}$ with $\theta_L < \theta_H$. Monopolist profits are thus:

$$\pi(q, T) = T - cq.$$

- This induces indifference curves like in G-6.1 where along any given q, the slopes of the indifference curves are equal due to quasilinearity in T. And as in G-6.2, the indifference curves have the Single Crossing Property.
- Thus the driving point of the model of SDPD will be that $MRTS_H > MRTS_L$.
- See G-6.3. Since $v(0) = 0$, we have the participation constraints, or individual rationality constraints (IR) as shown.
- If the θ's were observed by the firm, the solution would have the form:

$$\theta_L v'(q_L) = c, \; and \; \theta_H v'(q_H) = c.$$

  See G-6.4. It is clear that $q_H > q_L$ under this scheme because at a point like E, $\theta_H v'(q_L) > c$ while at F, $\theta_H v'(q_H) = c$, as required.
- What about a two part tariff? This is also drawn in G-6.4. A firm would charge a per unit fee equal to its marginal cost, and the fixed fee would be the vertical distance from the origin to the point where the tangent line hits the vertical axis as shown.
- Now suppose a monopolist does not observe types but instead only knows:

$$Prob\{\theta = \theta_H\} = \lambda.$$

- Suppose a single two part tariff is the only thing that can be offered. Will the contract displayed in G-6.5 be choosen? No, since neither the high now low type's IR is satisfied. There is money left on the table since utility is quasilinear in T, simply increasing T will raise monopolist profits, while keeping the quantity decision of the consumers the same.
- So maybe we would get a contract like G-6.6 where the low types IR is binding but the high types enjoy some surplus.
- Typically under a single contract, the price charged will be higher than the marginal cost of the firm.

- However this is STILL NOT OPTIMAL! There is also the incentive compatibility (IC) constraint of the high type to consider. If we allow for more complex (nonlinear) contracts, we will obtain the optimal SDPD solution.

- So consider (q, T) to be our mechanism space. This is sort of like the Walrasian Auctioneer argument where we determine the optimal set of contracts that maximizes the expected profit of the monopolist while satisfying individual rationality and incentive compatibility. We seek an equilibrium where everyone participates and both types are truthful about their types (ie, they have no incentive to lie). So the mechanism maps types into (q, T) space.

- Consider our usual 4 IR and IC constraints:

$$\begin{aligned} IR_L: &\quad \theta_L v(q_L) - T_L \geq 0 \\ IR_H: &\quad \theta_H v(q_H) - T_H \geq 0 \\ IC_L: &\quad \theta_L v(q_L) - T_L \geq \theta_L v(q_H) - T_H \\ IC_H: &\quad \theta_H v(q_H) - T_H \geq \theta_H v(q_L) - T_L \end{aligned}$$

- So the monopolist's problem is:

$$Max_{q_L, T_L, q_H, T_H} \ \{\lambda T_H + (1-\lambda) T_L - c(\lambda q_H + (1-\lambda) q_L)\},$$

subject to:

$$IR_L,\ IR_H,\ IC_L,\ IC_H.$$

- So consider a series of claims which we can use to eliminate a couple of the constraints:
  - Claim 1: If IC_H and IC_L bind, then $q_H = q_L$. This is called the Pooling Equilibrium. Proof: Consider the two binding IC constraints:

    $$\theta_L v(q_L) - T_L = \theta_L v(q_H) - T_H.$$

    $$\theta_H v(q_L) - T_L = \theta_H v(q_H) - T_H.$$

    Subtract:

    $$(\theta_H - \theta_L) v(q_L) = (\theta_H - \theta_L) v(q_H) \Longrightarrow q_L = q_H.$$

    QED.
  - Claim 2: If IR_H binds, then $q_L = 0$. Proof: Since IR_H binds:

    $$0 = \theta_H v(q_H) - T_H.$$

    By IC_H,

    $$\theta_H v(q_H) - T_H \geq \theta_H v(q_L) - T_L.$$

    Since $\theta_H > \theta_L$,

    $$\theta_H v(q_L) - T_L > \theta_L v(q_L) - T_L.$$

Which, taking the first and last line together, violates IR_L. QED.

- Claim 3: If IC_H does NOT bind, then IR_H does bind. Proof: Consider IC_L:

$$\theta_L v(q_L) - T_L \geq \theta_L v(q_H) - T_H.$$

And a nonbinding IC_H:

$$\theta_H v(q_H) - T_H > \theta_H v(q_L) - T_L.$$

Suppose IR_H does NOT bind (proceed by contradiction):

$$\theta_H v(q_H) - T_H > 0.$$

This clearly cannot be an optimum because the monopolist can simply raise T_H, still not violating the other constraints until IR_H does bind.

- So if we are in a separating equilibrium, IC_H and IC_L cannot both bind.
  - If IC_L binds, by claim 3, IR_H binds and by claim 2, $q_L = 0$. Thus, we sell only to the high type but this isn't very interesting.
  - If IC_H binds, IC_L does not bind. IR_L must bind.

- So at an optimal separating solution where both types participate, IC_H and IR_L must bind.

- Solving our two constraints for the payment:

$$T_L = \theta_L v(q_L).$$

$$T_H = \theta_H(v(q_H) - v(q_L)) + \theta_L v(q_L).$$

Substituting into the monopolist's problem we have:

$$Max_{q_L,q_H} \;\; \{\lambda(\theta_H v(q_H) - \theta_H v(q_L) + \theta_L)v(q_L) - cq_H) + (1-\lambda)(\theta_L v(q_L) - cq_L)\}.$$

FOC(q_H):

$$\lambda(\theta_H v'(q_H) - c) = 0 \Longrightarrow \theta_H v'(q_H) = c, \text{ (efficient)}.$$

FOC(q_L):

$$\lambda(-\theta_H v'(q_L) + \theta_L v'(q_L)) + (1-\lambda)(\theta_L v'(q_L) - c) = 0 \Longrightarrow v'(q_L)\left(\frac{\theta_L - \lambda\theta_H}{1-\lambda}\right) = c.$$

- See G-6.7. So the solution is represented graphically by E_L and E_H. The contract for the low type is on the IR_L curve and the one for the high type is on his IC_H curve. Note that IR_H does not bind so there is still surplus going to the high type, but not as much as in the case of a two part tariff where linear pricing is used.

- Note that for a low type, his optimal quantity in the separating equilibrium is smaller than the social planner solution (when types are observed). So think of a scenario where a low type comes into a store and purchases the low contract (q_L, T_L). He has just identified himself as a low type in our separating equilibrium world. Once this happens, the store owner should say, "Hey!, before you leave, I know there are still gains from trade between us so why don't you buy this much more of the good." And the consumer will do so. However, with a policy like this, high types would want to come in and fake being a low type to get the same deal. So an implicit assumption in all of this is a sort of "Sequential Credibility." The firms MUST be willing to let the low type walk out the door even though it is not efficient for him to do so. It is, however, optimal.

- This leads into a disussion of the Coase Conjecture next time.

7 Lecture 7: September 22, 2005

7.1 More on Price Discrimination and the Coase Conjecture

Mechanism Designer

- Recall the setup we had last time, where in order to satisfy the IR and IC constraints, we ended up in a situation where the high type consumers purchased their efficient quantity, but the low type ended up under-consuming. See G-7.1.
- The mechanism designer must be able to commit to allow a low type walk out the door as we described last time, knowing that further gains from trade existed. Allowing the low type to consume his efficient quantity would attract high types to that contract and the mechanism would break down.
- This leads us to the special case of the "durable goods monopolist."

A Durable Goods Monopoly

- Consider a monopolist who sells a good that is durable, ie, once it is purchased, those consumers are out of the market. A computer may be a reasonable example.
- On the first day the firm operates, the monopolist faces a demand curve like in G-7.2 and sets (p_1^m, q_1^m) to maximize profits as shown. Consumers with a willingness to pay greater than p_1^m purchase the good and then LEAVE the market. On the second day, the demand curve faced by the monopolist has essentially been truncated by those with the highest willingness to pay. It will be optimal for the firm to set a price like p_2^m and sell more of the good to the next segment of the market. This continues until all consumers have purchased the good and the firm has made crazy profits.
- But this clearly isn't sustainable, because the smart consumer with a high reservation price will want to "hold out" until later in the game when prices have fallen. The only way to avoid this would be for the monopolist to commit to only operate on day 1 and to throw away any remaining product that didn't sell. This idea motivated the Coase Conjecture.
- Suppose we have a durable goods monopolist who cannot commit to throwing away the extra amount of the good once the market has cleared on day one. Thus he can't avoid the situation where high reservation consumers will be inclined to hold out until tomorrow to buy. The **Coase Conjecture** said that profits for the monopolist would vanish in "the twinkling of an eye" as prices fell inexorably to marginal cost.
- Two sets of economists have studied this idea: Gul, Sonnenschein, and Wilson (GSW) and also Levin, Tirole, and Fudenberg (LTF). We study the model put forth by the former.

Single Monopolist with Constant Marginal Cost - GSW Model

- Without the loss of generality, assume marginal costs are zero. The monopolist produces are single durable good and there is a continuum of infinitely lived consumers, each of whom have inelastic demand for one unit of the good.
- A consumer of type v gets flow utility from her monetary holdings of $(1-\delta)$ and if she owns the good, also gets flow utility of $v(1-\delta)$ from the good.
- Both the buyer and seller discount future periods according to $\delta \in (0,1)$.
- Informational assumptions. In GSW, they assume a mass of consumers equal to 1. Consider a measure determining the distribution of consumers that can be described via a CDF, $F(\cdot) : [\underline{v}, \overline{v}] \mapsto [0,1]$. So $F(v) = Prob(V \leq v)$. [Note: In LTF, they model just one buyer and one seller where the type of the buyer is indexed by v. This leads to the same result as GSW.]
- Suppose a consumer buys a unit of the good in time t and at price p_t. Suppose she has initial money wealth, M. If she never buys the good, her utility is:

$$\overline{u} = \sum_{i=0}^{\infty} \delta^i (1-\delta) M = \sum_{i=0}^{t-1} \delta^i (1-\delta) M + \delta^t \sum_{i=0}^{\infty} \delta^i (1-\delta) M.$$

  If trade occurs at (t, p_t), then:

$$u = \sum_{i=0}^{t-1} \delta^i (1-\delta) M + \delta^t \sum_{i=0}^{\infty} \delta^i (1-\delta)(M - p + v).$$

  Which we can rewrite:

$$u = \sum_{i=0}^{t-1} \delta^i (1-\delta) M + \delta^t \sum_{i=0}^{\infty} \delta^i (1-\delta) M + \delta^t \sum_{i=0}^{\infty} \delta^i (1-\delta)(v - p_t).$$

  Thus, the net gains from trade at (t, p_t) are:

$$u - \overline{u} = \delta^t \sum_{i=0}^{\infty} \delta^i (1-\delta)(v - p_t) = \delta^t (1-\delta)(v - p_t) * \frac{1}{1-\delta}.$$

  Or,

$$u - \overline{u} = \delta^t (v - p_t).$$

- So, now lets turn to our CDF, $F(v)$. Consider a one period problem where the seller offers a price, p, and the buyer either accepts or reject. The seller's choice of a price determines the different subgames of the game. For any given choice of p by the seller, the buyer's strategy is to either accept if $v \geq p$ or reject if $v < p$. Thus, for any given p, $1 - F(p)$ of the buyers will buy. So $1 - F(p)$ is the demand faced by the monopolist.

He solves:

$$Max_p \{p(1 - F(p))\}.$$

- So now consider two possible forms of the CDF, F.
  - (1) Uniform CDF. $F(v) = v - \underline{v}$, with $\overline{v} = \underline{v} + 1$. Then demand at price p is:

    $$1 - F(p) = 1 - (p - \underline{v}) = 1 + \underline{v} - p.$$

  - (2) Binomial CDF. Suppose:

    $$F(v) = \begin{cases} 0, & v < 1 = \underline{v} \\ M_1, & v \in [1, 3) \\ 1, & v \geq 3 = \overline{v} \end{cases}$$

    See G-7.3.

- So from the perspective of an individual consumer of type v, her demand equals 1 if $p \leq v$ and equals 0 if $p > v$. See G-7.4
- So there are clearly many forms of F we could use, including a continuum of different types of consumers as in the uniform case. We will focus on the binomial setup.
- Thus consumers are of only two types, $v \in \{1, 3\}$. Claim: If $\hat{p}$ is optimal for the seller, then:

  $$\hat{p} \in \{1, 3\}.$$

  This is straightforward. For any price less than one, all consumers buy, but the firm can increase profits by raising the price up to one and still capturing the entire market. For a price between one and three, the low types are priced out but the firm can strictly increase profit by raising the price to three to capture only the high type, or lower the price to one to again capture the entire market. For any price above three, demand is zero. Specifically,

  $$\pi(p) = \begin{cases} p * 1 = p, & p \leq 1 \\ p * (1 - M_1), & p \in (1, 3] \\ 0, & p > 3 \end{cases}$$

- Thus, the monopolist should set $\hat{p} = 3$ if:

  $$3(1 - M_1) \geq 1 \Rightarrow M_1 \leq \frac{2}{3}.$$

  So set the high price as long as M_1 is not too big, ie, $1 - M_1$ (the demand by the high types) is not too small.

- See G-7.5 for a graphical depiction of the profit function. For prices between 0 and 1, it slopes up at a constant rate (of 1) and then at $p = 1$, it jumps down to $(1 - M_1)$ (as we have priced out the low type) and then slopes up at a constant rate (of $1 - M_1$). So the problem is: which is higher, point A or point B?

8 Lecture 8: September 27, 2005

8.1 More on Coase and Durable Goods Monopoly

- Recall our setup from last time where the monopolist's problem reduced to:

$$Max_p \{p(1 - F(p)\}.$$

FOC:

$$1 - F(\hat{p}) - \hat{p}f(\hat{p}) = 0 \Longrightarrow \hat{p} = \frac{1 - F(\hat{p})}{f(\hat{p})}.$$

- Since the optimal price is determined by the marginal consumer who is willing to pay for the good, we could replace the optimal price with the valuation of that last consumer:

$$1 - F(\hat{v}) - \hat{v}f(\hat{v}) = 0 \Longrightarrow \underbrace{(\hat{v} - \frac{1 - F(\hat{v})}{f(\hat{v})})}_{Virtual\ Utility} f(\hat{v}) = 0.$$

- Recall from last time, as $M_1 \to 0$, ie, the fraction of high types got smaller and smaller, it was optimal to sell only to the high types. See G-8.1. By Coase, it was also necessary for the monopolist to be able to commit to throwing away any remaining product after the initial sale. This is only a problem though if a high price is optimal in period 1. If the low price is immediately optimal, then the market is open at $p = 1$, everyone buys, and the market has cleared.

8.2 Infinite Horizon Durable Goods Monopolist

- The firm, or seller, can either commit to an entire sequence of prices and then let the market decide how much to demand at all periods, or he could reoptimize prices every period. It turns out that by tying his hands, the seller is actually better off. We flesh this out below.

- **Lemma 1** Suppose a buyer with valuation v buys in period t and at price p. Another buyer with valuation $v' > v$, buys in period t' and at price p'. Then:

$$t' \leq t, \text{ and } p' \geq p.$$

Proof: The following two equalities must hold for the high type and low type:

$$\delta^{t'}(v' - p') \geq \delta^t(v' - p).$$

$$\delta^{t'}(v - p') \leq \delta^t(v - p).$$

Subtract:

$$\delta^{t'}(v' - p' - v + p') \geq \delta^t(v' - p - v + p).$$

$$\delta^{t'}(v' - v) \geq \delta^t(v' - v).$$

$$\delta^{t'} \geq \delta^t.$$

$$t' \leq t$$

And since it must be that prices are either stable or decreasing over time, $p' \geq p$. QED. This is like an incentive compatibility constraint in the infinite horizon case.

- Note the lemma above holds for a seller who precommits or one that does not commit to a price sequence. Either way, as time goes by, the segment of the market that has not yet purchased is becoming more and more concentrated with low types. There should be an incentive for the seller to lower his price in response.

- The important part is this: With commitment, the seller might choose to ignore the later periods where all that is left is low types to sell to. Since the seller can solve out his optimal price sequence under both commitment and no-commitment (assuming he knows the distribution of types), the no-commitment strategy of prices is still available to him under the commitment regime. Hence commitment must be better by revealed preference because this regime has the added benefit of the buyers knowing that in some future period, the seller cannot renege on his price plan.

- **Lemma 2** Denote the price sequence in the no-commitment game as follows:

$$p_t = p_t\Big((p_0, q_0), (p_1, q_1), \ldots, (p_{t-1}, q_{t-1})\Big) \equiv p_t(h_t),$$

or my price choice is conditional on the entire history of prices and quantities that have thus far been observed. Then in any subgame perfect equilibrium (SPE) of the game, with or without commitment, $p_t \geq 1$, ie the price is always set to be at least as big as the valuation of the lowest type.
Proof. Denote:

$$p^* = inf\ \{p|\text{ seller offers } p \text{ after some history, } h_t, \text{ in some SPE}\}.$$

So p^* is the lowest possible price in ANY SPE of the game. Suppose we play an equilibrium, σ, and reach a history, h_t, where p^* is to be offered. Proceed by contradiction. Assume that $p^* < 1$. Let,

$$p' = p^* + \epsilon < 1.$$

If p' is offered, the lowest type consumer will accept if and only if:

$$\delta^t(1 - p') = \delta^t(1 - p^*) - \delta^t\epsilon \geq \delta^{t+1}(1 - p^*).$$

So he either gets p' today, or in the very best alternate situation, he waits one period and gets the best possible price, p^*. If the lowest possible type consumer accepts, it must be that EVERYONE accepts this price. This cannot be optimal for the seller, so it must be that he rejects, or:

$$\delta^t(1 - p^*) - \delta^t\epsilon \leq \delta^{t+1}(1 - p^*).$$

$$1 - p^* - \epsilon \leq \delta 1 - \delta p^*.$$

$$1 - \delta - p^* + \delta p^* \leq \epsilon.$$
$$\underbrace{(1-\delta)}_{>0}\underbrace{(1-p^*)}_{>0} \leq \epsilon.$$
$$\epsilon > (1-\delta)(1-p^*) > 0.$$

Thus ϵ is bounded away from zero. Since this is true, then there exists an epsilon, say $\tilde{\epsilon}$, that will violate this condition (that ϵ is bounded away from zero) which will make the entire market buy at that price. Consider:

$$\tilde{p} = p^* + \tilde{\epsilon}, \;\; \tilde{\epsilon} = \frac{1}{2}(1-\delta)(1-p^*).$$

Clearly,

$$\tilde{\epsilon} < (1-\delta)(1-p^*),$$

and thus, the entire market will buy at $\tilde{p}$. Thus, p^* cannot be optimal if $p^* < 1$. Note if $p^* = 1$, then our condition collapses to:

$$\epsilon > (1-\delta)(1-\underbrace{p^*}_{1}) = 0,$$

which is true for all epsilon. QED.

- **Proposition for Commitment** Let $\overline{p} = \{p_t\}_{t=0}^{\infty}$ be the optimal sequence of prices for a seller that can commit to prices. Then given our binomial type setup from before, the solution looks like:

$$p_t = 3 \; \forall \; t, \;\; or \;\; p_t = 1 \; \forall \; t,$$

depending on which of the two prices is optimal in the static one period take it or leave it game.
Proof: We don't prove this here but the idea is this: suppose you assumed the optimal strategy would be for the seller to set $p_t = 3$ for a long time, exhaust the market for high types, and then lower the price. If the high types discounted the future enough, they wouldn't wait for the lower price and the low types could do no better than waiting. However, given this is a dynamic game, the high types get utility of zero from buying at price three so they are strictly better off by waiting. So how about a high price of 2.95 and then lowering the price after a sufficiently long period of time to dissuade the high types from waiting, but then gaining the rest of the market at the low price. This also turns out not to be optimal due to the loss in profits from lowering the price given to the high types. The proof is in Vincent's notes. It comes more out of the math than it is an intuitive result.

- Since the seller wants to set the highest price possible, it makes sense that the price sequence will satisfy:

$$(3 - p_t) = \delta(3 - p_{t+1}),$$

or the high type is just indifferent between buying and not buying. For $\delta = \frac{1}{2}$, this implies:

$$p_t = 2.75,\ p_{t+1} = 2.5,\ p_{t+2} = 2,\ p_{t+3} = 1,$$

where the high types buy in the early periods and the low types buy later on. Over time, more weight is placed on the low types (because they are all that's left in the market), and $p_t \to 1$ as $t \to \infty$. The price path starts high and gradually falls. If $1 - M_1$ is large (ie, there are a lot of high types), the game last longer and the starting price is large (closer, but not equal, to three). We will show the game will always end in a FINITE number of periods.

9 Lecture 9: September 29, 2005

9.1 Durable Goods Monopolist: No-Commitment Regime

- The monopolist faces a lack of strategic power compared with the commitment regime. We want to address just how big of a constraint this is on the firm.

- Note that as more and more low types make up the market, the gains from having the information about consumer's types is smaller. In fact, for a fairly large range of relative proportions, we can nail down the optimal price response by the monopolist: set a low price and capture the entire market. Game over.

- So first we find the point at which it is optimal to switch to the low price, and then we slowly work backwards by adding more and more high types. If there are lot of high types initially, the price set in the first period will be higher. See G-9.1. Note the strategy turns out to be the same if demand is stepping down like our binomial setup, or if it is a continuous, smooth, decreasing function.

- **Theorem** Under a Lipschitz (technical) condition on F, and if $\underline{v} > c$, then there exists a unique SPE price path under the no-commitment regime, such that prices are deterministic (the monopolist does not randomize), and for every possible discount factor, δ, there exists a $T(\delta)$ such that the game ends by period T. So even with a ba-zillion of infinitely lived consumers, the games ends in a finite number of periods.

- **Remark** The final price would have to be $p_T = \underline{v}$ and the price path must be strictly decreasing. This is because in every period, a strictly positive proportion of consumers will buy the good.

- **Remark** Is it plausible that there is this gap between the lowest valuation consumer and the marginal cost of the firm? Ausubel and Deneker say no. The distribution should really look like G-9.2.

- Now suppose that the "period" in question is Δ minutes. What happens as Δ goes to zero. If the waiting time between subsequent offers by the firm gets small, my discount rate of waiting until the next period to buy, δ, gets closer to one. Ie, I discount waiting less. We might posit that the relationship is:

  $$\delta = e^{-r\Delta t},$$

  where t is the number of periods. As $\Delta \to 0$, $\delta \to 1$. And as $\delta \to 1$, $T(\delta) \to \infty$. So the upperbound on the length of the game gets larger.

- But from Coase's perspective, the total length of the game, $\Delta * T(\Delta)$ may have a different result. In fact, he shows:

  $$lim_{\Delta \to 0}\, \Delta * T(\Delta) \to 0,$$

  So Δ goes to zero faster than $T(\Delta)$ goes to infinity. This again, is Coase's idea that the rents from the monopoly vanish in the twinkling of an eye. Price ends up at the low price immediately, the market clears, the monopolist makes no rent, and the game is over.

- So the idea here is that the monopolist (though she is the only firm in the market) is competing again her "future selves." As the time between offers gets smaller, the good she offers today versus the one she offers tomorrow become more homogeneous, and then the market becomes effectively more competitive. This drives profits to zero.

- From a bargaining perspective, this is unsatisfying because it seems that unless we can rationalize a certain "waiting period" between offers, the market for a durable good should only be open for one period and monopoly in question really has NO monopoly power. So as a modeller, you need to worry about the market institutions which are in place. Maybe there really is a reason why when I make my offer today, there is something preventing me from immediately making another offer once the consumer(s) decide to buy or not.

- **Model Extension 1** Consider the leasing of durable goods. While in our original model, we had that the consumers who purchased the good go out of the market forever, if instead you only rented the good, that consumer would eventually re-enter the market next period and the demand is essentially stable. Thus the optimal rental price is the monopoly price. This avoids the problem of sequential rationality as the firm is not competing with its future selves. The counter argument to why ALL durable goods monopolist don't engage in leasing is the classic principal/agent type issues with moral hazard and adverse selection.

- **Model Extension 2** Consider a market where every period, the monopolist sets his price and the high types (or a proportion of the high types) buy and then the market is truncated. But now assume there is some exogenous entry (births) of agents in the model every period. We might see the price progression as we noted above with the price falling in each period, but eventually the mass of high types will grow again and the monopolist will find it optimal to set the high price. Thus we get a sort of cyclical nature to prices.

- **Final Remark** Recall in the lemma from lecture 8, we had that an agent of type v would buy at price p in time t iff:

$$\delta^t(v-p) \geq \delta^{t'}(v-p') \ \ \forall \ (t',p') \neq (t,p).$$

We could also think of offers in (p,π) space instead of (p,t) space where π is the probability of a transaction happening in a given period. Thus, we compare:

$$\pi(v-p) \ \ vs \ \ \pi'(v-p').$$

Note π and δ^t are both in the interval $(0,1)$ so mathematically, these are very similar problems. See the problem set question for more on this.

9.2 Product Differentiation

- What does it mean to have close but NOT identical products? No metric really exists beyond cross price elasticities. Some relationships may appear clear (apples versus car tires), while others are more subjective.

- Two main types of Product Differentiation.

  - (1) Vertical Differentiation (VD) The main example of this might be product quality. If given two goods A and B with $P_A = P_B$, A is vertically better than B if everyone prefers A to B. Typical preferences might be:

    $$v(s, p; \theta) = \theta s - p,$$

    where s indexes quality, p is the price, and θ are private preference parameters. Consumers still differ with respect to tastes, but EVERYONE thinks that more s is better than less.

  - (2) Horizontal Differentiation (HD) Here we have heterogenous tastes/consumers across products. So maybe IBM versus Macintosh. The classic method of distinguishing types is the hotelling model of a horizontal line (the beach) with two firms (ice cream stands) at $x = 0$ and $x = 1$. Transportation cost is t and utility from an ice cream cone (chocolate) is S. Then the equation of the marginal man would be:

    $$S + tx - p_0 = S + t(1 - x) - p_1.$$

    $$x = \frac{p_0 - p_1}{2t} + \frac{1}{2}.$$

- A nice example to distinguish VD from HD is as follows: A car with standard options and one with A/C are vertically differentiated. A car with A/C and one with a spoiler are horizontally differentiated.

10 Lecture 10: October 4, 2005

10.1 Bundling by a Monopolist

- Bundling is the act of creating a new product by bundling goods together. Virtually all goods we purchase are part of a bundle. Cars are bundles of parts and return airline tickets are bundles of one way tickets.
- There may be a technological reason for the bundle, but the more interesting case, economically, will be when the bundle creates a more efficient way to extract rents.
- Bundling is a form of tying and there are two types of tying:
  - (1) Tie-Ins: Consumer can only buy product A if they buy B as well. Eg, prix-fixe meals, razor blades bundled as a 4-pack.
  - (2) Tie-Outs: Consumer can only buy A if they agree to buy B from the same seller. Eg, Printers and cartridges where the replacement cartridges are patented, razor blades replacements that only fit the one brand.
- There is no explicit antitrust law prohibiting tying especially when there is a technological reason for the bundle.
- Consider two goods, A and B, supplied by a monopolist. The firm has three pricing options.
  - (1) Offer (P_A, P_B). Unbundled.
  - (2) Offer (P_{AB}). Pure Bundling.
  - (3) Offer (P_A, P_B, P_{AB}). Mixed Bundling.

  A mixed bundle like $(1, 1, 3)$ is superadditive. This might make sense for a car where you could theoretically buy the parts for cheaper and assemble but the firm can charge a higher price for the finished product. This might present an arbitrage problem with people buying the parts, making the car, and reselling. A mixed bundle like $(2, 2, 3)$ is subadditive. Round trip airline tickets are a good example. Unbundling must not be possible. Firms (microsoft) make it difficult to unbundle their products in order to maintain this pricing strategy.
- If there are heterogenous agents, it may be optimal to offer a bundle in addition to the individual products to extract more of the consumer's surplus.

Stigler "Block Booking" Example

- See also Adams/Yellen, QJE. An example of block booking is VISA's "honor all cards rule." Merchants that accept VISA cards must accept ALL cards, even those that cost relatively more for the merchant to accept.

- Consider a production studio offering their lines of movies to movie houses. They might decide to require that even though some films will be blockbusters and some will fail, they must buy and show all the movies in the bundle. Consider two types of movie houses with valuations:

Type I: Value of Blockbuster = 4, Value of Bad Movie = 1.

Type II: Value of Blockbuster = 3, Value of Bad Movie = 2.

- If the firm wanted to sell all the good and bad movies to all the movie houses, they would have to set a price for the blockbusters of 3 and for the bad movies of 1. This yields revenue of 4.
- If the firm instead offered only a bundle of the movies, they could sell at a price of 5, yielding revenue of 5. The production studio is strictly better off offering only the bundle.
- **Remark** Note that the types are heterogenous wrt to individual films, but they are homogenous wrt to the bundle. Since there is negative correlation among the types, the market become fairly homogenous when dealing with the whole bundle. The bundle turns out to be the superior rent extraction device in this setting.

Metering

- Bundling may be optimal if metering is possible. Consider a single indivisible product which is complimentary to a divisible product which is produced competitively. Maybe the firm makes the computer but the complimentary good are the old computer cards that must be fed into the computer (in 1922).
- Assume consumers are privately informed about their values of the products. The firm produces one unit of A (indivisible) and then the agents purchase q units of B to go with their unit of A. Consumer preferences for B:

$$\theta v(q) + M, \quad \theta \in \{\theta_H, \theta_L\}, \quad v' > 0, v'' < 0.$$

- Suppose good B is produced competitively at marginal cost c. The monopolistic producer of good A will want to the price of good B to be as low as possible, namely marginal cost. If he could produce both, we would charge a two part tariff with the entry fee being the price of A and the per unit cost of B being c.
- The monopolist needs either a contractual tie-out, or a design based tie-out to make sure the consumer must use good B with good A. In other words, they need to be able to METER the purchase of B.
- So the next topic addresses whether it is necessary for the goods in the bundle to be complimentary or not.

Monopolist Bundling Model

- So we are moving towards a model as follows: a monopolist sells two goods to privately informed consumers with unit demand for each of the two goods. A consumer is of type $v = (v_1, v_2)$. Suppose the consumer pays a price of p for some bundle. Then
  - (i) if the consumer gets only good A, his utility is:
  $$u = v_1 - p$$
  - (ii) if the consumer gets only good B, his utility is:
  $$u = v_2 - p$$
  - (iii) if the consumer gets both A and B, his utility is:
  $$u = v_1 + v_2 - p$$
- Our first step wil be to solve the one good model and then move to the two good model. More next time.

11 Lecture 11: October 6, 2005

11.1 More Bundling by a Monopolist

- Consider again a monopolist that produces two goods at zero marginal cost. The informed consumer is of type (v_1, v_2) and if he buys at price p, he gets:

$$u(p, q, v) = v_1 q_1 + v_2 q_2 - p, \quad q_i \in \{0, 1\}.$$

- The consumer observes v but the monopolist only knows $F(v)$. Assume $v \in [0, 1]^2$.
- A few examples of $F(v)$:
  - (1) $v_2 = 1 - v_1$, $v_1 \sim f(v_1)$.
  - (2) $F(v_1, v_2)$ with $f(v_1, v_2) > 0$.
  - (3) $F(v) = F_1(v_1)F_2(v_2)$, independent.
  - (4) $F(v) = F(v_1) * F(v_2)$, iid.
- So suppose $F(v)$ is iid which induces pdf, $f(v_1, v_2) = f(v_1)f(v_2)$. Assume $f(x) > 0 \; \forall \; x \in [0, 1]$ and f is continuous.
- Also assume:

$$(x - \frac{1 - F(x)}{f(x)})$$

is strictly increasing. This is called the Monotone Hazard Rate Condition (MHR).

Part I: Monopolist Sells Only Good 1

- See G-11.1 for the valuations of a consumer. Note that v_2 is irrelevant when determining if a consumer will buy good 1.
- The monopolist sells a single good at price p_1. The buyer accepts if $v_1 \geq p_1$. The buyer accepts with probability $1 - F(p_1)$. Thus the monopolist's problem is:

$$Max_p \; p_1(1 - F(p_1)).$$

FOC:

$$1 - F(\hat{p}_1) - \hat{p}_1 f(\hat{p}_1) = 0.$$

Note this holds with equality because the monopolist would never set a price of zero. We can rewrite this as:

$$-(\hat{p}_1 - \frac{1 - F(\hat{p}_1)}{f(\hat{p}_1)})f(\hat{p}_1) = 0.$$

Buy our assumption, the term in parens is increasing, $f(\hat{p}_1) > 0$, so the whole thing is decreasing. See G-11.2. Note this means there is a single, unique $\hat{p}_1$ that solves.

- We can pose the problem a bit differently: Suppose the marginal consumer who buys has valuation $v = \hat{p}_1$. The monopolist's problem is thus:

$$Max_v \; v(1 - F(v)).$$

Consider the total differential:

$$(1 - F(v))dv - vf(v)dv.$$

So the first term is the gains from the monopolist raising its price above $v = \hat{p}_1$ (by receiving the higher price from those that continue to buy) and the second term is the loss in consumers who now find it optimal not to buy. See G-11.3.

Part II: Monopolist Sells Both Goods

- Assume the buyer's valuations are still statistically independent. This rules out the Stigler case there valuations are negatively correlated and this alone makes bundling optimal. The monopolist still faces zero costs.

- The firm names prices (p_1, p_2, p_{12}), ie a mixed bundle. Note this includes the possibility of pure bundling as well as independent goods pricing by setting the other prices to infinity.

- We will show that mixed bundling weakly dominates pure bundling and independent goods pricing.

- Suppose the firm names (p_1, p_2) only and not a bundle price. Then consumers buy as in G-11.4. Profits are:

$$\pi(p_1, p_2) = (1 - F(p_1))p_1 + (1 - F(p_2))p_2.$$

Thus we have two isolated problems and the same analysis as above results. We get a unique $(\hat{p}_1, \hat{p}_2)$ which solves the monopolist's problem. Setting the independently optimal price for each good is optimal.

- Suppose now only a bundle is offered, p_{12}. Then buyers as in G-11.5 will buy, ie if $v_1 + v_2 > p_{12}$.

- Now suppose a mixed bundle is offered of $(\hat{p}, \hat{p}, 2\hat{p})$. See G-11.6. Those in the region I and II as well as those that buy the bundle are clear. Now consider those buyers with valuations in the region labled (*). Consider the bottom right region. Those buyers would be willing to pay $\hat{p}$ for good 1 but not for good 2. With the bundle added as an option, they buy the good 1 only if:

$$v_1 - \hat{p} > v_1 + v_2 - 2\hat{p} = v_1 - \hat{p} + \underbrace{v_2 - \hat{p}}_{negative}.$$

So this is always true! Thus they continue buying only good 1 and the same is true for the upper left region. Thus the pricing scheme $(\hat{p}, \hat{p}, 2\hat{p})$ leaves the firms just as well off as pricing goods independently.

- Now consider G-11.7. If we bump up $\hat{p}_1$ by just a bit, some consumers get priced out like we mentioned above in the single good case. However, there is a region (labled on the graph), where consumers change from buying a single good to buying the bundle. Thus the typical solutions looks like G-11.8. A characteristic of the solution is that the bundle is subadditive:
$$\hat{p}_{12} < \hat{p}_1 + \hat{p}_2.$$

- It can also be shown that mixed bundling is weakly better than pure bundling.

- **Remark** This wasn't very clear but ... Vincent goes on to say that if instead of offering take it or leave it offers, you instead offer probabilities of sales, the firm can do even better. Maybe something like: buy good 1 at price p_1 today and I'll give you a 70 percent chance of buying good 2 from me tomorrow at price p_2. We need some assumptions on the taste distributions (independence, MHR, etc), but this works against the Myerson result that the optimal pricing scheme in the multi-good case is the same as in the independent goods case (ie, you find the optimal prices in each independent market and then just offer all those prices in the multi-good setting). Givens.

12 Lecture 12: October 11, 2005

12.1 More on Bundling

- Here we look for an efficiency motivation for bundling by a firm.
- This is the Matthewson/Winter idea of tying as a response to demand uncertainty. We consider a "tie-out" contract, also known as a "requirements-contract."
- Consider Exxon supplying Bob's Big Gas and Beef Jerky with gasoline. Suppose they say, buy this one tank from me today at a certain price but you have to buy all your future gas from me as well at some other price. This ties-OUT BP and shell for example.
- Model. Assume:
  - (1) A firm is a monopoly in good 1, produced at cost, c_1. Good 2 is produced competitively at cost, c_2.
  - (2) The market of consumers are identical with Q-linear preferences generating utility function:
  $$u_1(Q_1) + u_2(Q_2) + e,$$
  where e is all other goods. This generates a quasi-convex indirect utility function of the form:
  $$v_1(p_1) + v_2(p_2) + e,$$
  which in turn generates demands:
  $$Q_1 = v_1'(p_1), \quad Q_2 = v_2'(p_2).$$
  - (3) The monopolist fixes a pair of prices (p_1, p_2) where $p_2 > c_2$ is enforced by a tie-out. We might think the monopolist will cut a break on p_1 in exchange from getting the buyer's business on good 2 at p_2 (strictly higher than the competitive price).
  - (4) Suppose there exists a choke price, $\overline{p}_1 \ni Q(\overline{p}_1) = 0$.
- **Proposition** The monopolist's optimal price pair has the following property:
  $$\frac{p_i - c_i}{p_i} = \frac{K}{\epsilon_i},$$
  where ϵ_i is the market i elasticity of demand and K is a positive constant (same across markets). So note $p_2 > c_2$. Also, $p_1 < p_1^m$ if $K < 1$.
- See G-12.1. Suppose we start at a point like E at the monopoly price in good 1 and the competitive price in good 2. Since the consumer's indifference curves are concave and the isoprofit lines are convex, there are gains to moving to a point like F which means selling below the monopoly price and above the competitive price in markets 1 and 2.

Because the firm controls the sales of good 1 (completely), if the buyers really wanted good 1, then the firm could, in fact, do even better by threatening not to sell good 1 at all, resulting in the same utility as the choke price would. From that point, we can repeat the analysis above and again move up to a point with very high profits for the monopolist created with (possibly) a price below the monopoly price in the market for good 1 and a price above the competitive price in market 2. The monopolist achieves a bliss point.

- The idea is that the monopolist's ability to take good 1 off the table provides him with a strategic power. He leverages his monopoly in market 1 into the competitive market for good 2. Since the social losses in raising the price in market 2 above marginal cost are very small compared with the social gain from the monopolist selling at something below the monopoly price, it may be that from a social planner/efficiency point of view, bundling (tying) is welfare improving.

- However(!), a 2-part tariff still does better. There may be institutions which prevent 2 part tariffs, but if there were allowed, the monopolist could simply extract all surplus and bundling is no longer the optimal solution. It removes the efficiency gains from bundling.

- Even if 2-part tariffs are illegal in market 1, as long as the monopolist is present in market 2, the motives for bundling vanish. Consider charging $p_1 = c_1$ and tie-in good 2. Force the consumer to buy from the firm in market 2 at $p_2 = CS_1^c + c_2$, where CS_1^c is the consumer surplus in market 1 from the competitive outcome. This mimics the 2-part tariff result.

- So why do we still see requirements contracts? Matthewson/Winter say that if there is a statistical relationship between the demand for good 1 and good 2 (ie, they may be positively correlated), then the optimal result is to impose a tie. This is a strange and messy result which comes out of some informational assumptions. It clearly doesn't crank any tractors for Vincent.

13 Lecture 13: October 13, 2005

13.1 Oligopoly Theory

- Recall the general market structure ranging from monopoly (price making) behavior to perfect competition (price taking) behavior. Oligopoly falls somewhere in the middle. In these models, the price formation mechanism is much more complicated. Firms know they and their competitors have some power over price.

- The idea that there is one firm in a monopoly and an infinite number of firms under perfect competition doesn't hold up rigerously. A large number of firms is a necessary, but NOT sufficient, condition for competitive pricing.

Outline of Oligopoly Lectures

  - (1) Description of the classic Cournot game.
  - (2) Novshek's Existence result.
  - (3) Friedman's Uniqueness result for n firms.
  - (4) Limit results for Cournot games.
  - (5) Bertrand games and existence issues.
  - (6) Differentiated products Bertrand (due to Vives).
  - (7) How do you reconcile Cournot and Bertrand (Kreps and Schenkman).

1. Description of the Classic Cournot Game

- Cournot's 1838 book was the first entrance into this literature and focused on quantity competition among firms. Bertrands 1883 response to Cournot's book really got people talking as he focused on price as the choice variable. This led to COMPLETELY different results. Nash, 1950, drives the research to a new level with the idea of mutually best responses.

- Consider Cournot's classic game. We have players (firms), $i = 1, \ldots, n$.

- The strategies of the firms are output choices:

$$x_i \in S_i = \Re^+,$$

  where S_i is the strategy space. Note there is an INFINITE set of strategies.

- Define aggregate output:

$$x = \sum_{i=1}^{n} x_i.$$

- Define the payoffs (profits) to the players (firms) as:

$$\pi_i(x_i, x_{-i}) = x_i p(x) - c_i(x_i),$$

where $p(x)$ is the market demand with $p'(x) < 0$ and $c_i(x_i)$ is the firm specific costs.

- So the clear weakness of this setup is: how does x get translated into a market price? We have sort of a black-box and we just have to live with this for now.

- What's our solution? A Nash Equilibrium (NE) is a profile of outputs, $x^* = (x_i^*, x_{-i}^*) \in \Re^n$, such that:

$$\forall\ i,\ \pi_i(x_i^*, x_{-i}^*) \geq \pi_i(x_i, x_{-i}^*).$$

We can also write the solution in a different way. Define firm i's best response function as:

$$BR^i(x_{-i}) = arg\ max_{x_i}\ \pi^i(x_i, x_{-i}^*),$$

and,

$$BR(x) = (BR^1(x_{-1}), BR^2(x_{-2}), \ldots, BR^n(x_{-n}))'.$$

Then a NE is a fixed point of BR:

$$x^* \in BR(x^*).$$

- So recall our Nash Existence theorem from 601 which says that as long as the strategies are finite, the payoff functions continuous, etc, we will always have a NE (possibly in mixed strategies). Clearly these assumptions fail here. Consider a firm with fixed costs and convex variable costs. The non-convexities introduced makes the cost function discontinuous. So adding these assumptions (needed for Nash Existence) are strong and unrealistic.

- **Remark** A good example of a game with no solution. Consider a firm choosing $x \in [0, 1]$ with payoff $\pi(x) = x$, for $x \leq 1$ and $\pi(x) = 0$, otherwise. Clearly, the game has no solution. The firm wants to set $x = 1$ but that gives him a payoff of zero.

- **Definition** A function, $f : \Re_+ \mapsto \Re$ is continuous from the left if $\forall\ \epsilon > 0$ and $\forall\ \hat{x}$,

$$lim_{\epsilon \to 0}\ f(\hat{x} - \epsilon) = f(\hat{x}).$$

See G-13.1.

- **Theorem** Novshek. Consider the classic Cournot game. Suppose:

  - (1) Costs are continuous from the left and increasing.
  - (2) $p(x)$ is continuous and c^2 with $p'(x) < 0$.
  - (3) $xp(x)$ is bounded and $\exists\ z^*$ such that $p(z^*) = 0$.
  - (4) $xp''(x) + p'(x) \leq 0\ \forall\ x \in [0, z^*]$.

  Then there exists a pure strategy NE.

- **Remark** So note that fixed costs and even non-convex variable costs are ok. Assumption 3 bounds output to be less than z^* and assumption 4 means the profit function is quasi-concave (MR is downward sloping).

14 Lecture 14: October 18, 2005

14.1 Classical Cournot Model

- Consider G-14.1, a graph of 2 best response functions in the Cournot game. By Novshek, $\exists$ a solution, $x^* = (x_1^*, x_2^*)$, such that,

$$x_1^* = BR_1(x_2^*), \quad x_2^* = BR_2(x_1^*).$$

So Novshek gives us existance, but we could, as in the graph, still have multiple equilibria. Next we move to a uniqueness result.

3. Friedman's Uniqueness Theorem

- **Theorem** Suppose a NE exists (by Novshek). If,

$$\left|\frac{\partial^2 \pi^i}{\partial x_i^2}\right| \geq \sum_{j \neq i} \left|\frac{\partial^2 \pi^i}{\partial x_i \partial x_j}\right| \; \forall \, x, i$$

then the NE is also UNIQUE.

- Consider the case of two firms where I use superscripts to index the firm and subscripts to index partial derivatives. Given the profit function for firm 1: $\pi^1(x^1, x^2)$, at a NE, the FOC implies:

$$\pi_1^1(x^1(x^2), x^2) = 0.$$

Differentiating with respect to x^2,

$$\pi_{11}^1 * x_1^1(x^2) + \pi_{12}^1 = 0.$$

Or,

$$x_1^1(x^2) = -\frac{\pi_{12}^1}{\pi_{11}^1}.$$

So, on the LHS we have the inverse of the slope of firm 1's reaction function. π_{11}^1 is negative as we are maximizing profits. What about π_{12}^1 ? Consider the profit function for firm 1:

$$\pi^1 = P(x^1 + x^2)x^1 - c^1(x^1).$$

FOC:

$$\pi_1^1 = P'(x^1 + x^2)x^1 + P(x^1 + x^2) - c_1^1(x^1).$$

Cross partial:

$$\pi_{12}^1 = P''(x^1+x^2)x^1+P'(x^1+x^2) = \underbrace{P''(x^1 + x^2)(x^1 + x^2) + P'(x^1 + x^2)}_{\leq 0 \text{ by } Novshek} - P''(x^1+x^2)x^2. \quad (*)$$

So if $P''(\cdot) > 0$, then $\pi^1_{12} \leq 0$ by the second part of (*) and if $P''(\cdot) < 0$, then $\pi^1_{12} \leq 0$ by the first part of (*). Thus overall,

$$x^1_1(x_2) = -\frac{\overbrace{\pi^1_{12}}^{<0}}{\underbrace{\pi^1_{11}}_{<0}} \leq 0.$$

- And similarly for $x^2(x^1)$. So the result looks like G-14.2. Both reaction functions are downward sloping in (x^1, x^2) space.
- So consider the the slopes of the reaction curves for firms 1 and 2:

$$Slope(RC^1) = \frac{1}{x^1_1(x^2)} = -\frac{\pi^1_{11}}{\pi^1_{12}}.$$

$$Slope(RC^2) = x^2_1(x^1) = -\frac{\pi^2_{12}}{\pi^2_{22}}.$$

Thus,

$$\frac{1}{x^1_1(x^2)} + 1 = -\frac{\pi^1_{11}}{\pi^1_{12}} + \frac{\pi^1_{12}}{\pi^1_{12}} = \underbrace{\frac{1}{\pi^1_{12}}}_{\leq 0} \underbrace{\left[\pi^1_{12} - \pi^1_{11}\right]}_{?}.$$

Under Friedman's uniqueness theorem, (?) is non-negative. Thus,

$$\frac{1}{x^1_1(x^2)} + 1 \leq 0 \Longrightarrow \frac{1}{x^1_1(x^2)} \leq -1.$$

So firm one's reaction curve is steeper than firm two's (and downward sloping).

- Similarly for firm 2:

$$x^2_1(x^1) \geq -1.$$

- So the result is G-14.3. A unique NE.
- **Remark** Note that the Friedman theorem buys us uniqueness AND stability! If the slopes were switched around, then the equilibrium would be unstable.
- **Remark** Note also that $\pi^1_{12} \leq 0$ implies games in strategic substitutes. Ie, my marginal return to my choice of x is decreasing in your choice of x. In the Bertrand model, we'll reverse this.
- So consider firm i choosing x_i and denote total quantity as $X = \sum_{i=1}^n x_i$ and $X_{-i} = X - x_i$ as total quantity less firm i's quantity. Thus,

$$\pi^i(x_i, x_{-i}) = P(x_i + \sum_{j \neq i} x_j)x_i - c_i(x_i) = P(x_i + X_{-i})x_i - c_i(x_i).$$

So,

$$\tilde{\pi}^i(x_i, X_{-i}).$$

So we can write the profit function in terms of x_{-i}, a vector of everyone else's quantities. Or we can write it as a function of X_{-i}, total quantity by everyone else. Given this, the same analysis for the $N = 2$ case holds for the general case of N firms. Under Friedman's condition, in (X_{-i}, x_i) space, the slope of firm i's response curve is in the interval $[-1, 0]$. See G-14.4.

- Now consider a special case of linear demand: $P(X) = A - BX$ and $C_i(x) = c_i x_i$. It is easy to show that Novshek and Friedman both hold so $\exists!$ NE (ie, there exists a unique NE).

4. Limit Results for Cournot Games

- Suppose Novshek and Friedman hold. Suppose the best response functions are differentiable. The FOC for firm i:

$$x_i^* P'(X) + P(X) - C_i'(x_i^*) = 0.$$

Or,

$$-\frac{x_i^*}{X}\frac{XP'(X)}{P(X)} = \frac{P(X) - C_i'(x_i^*)}{P(X)}.$$

Or,

$$\frac{s_i}{\epsilon} = \text{markup}.$$

See problem set 5. So firms with a lower marginal cost will have a higher market share. This seems intuitive.

- Back to our linear case above and suppose $c_i = 0 \; \forall \; i$ and $A = B = 1$ and $N = 2$. See G-14.5. The total monopoly quantity is $\frac{1}{2}$. The total cournot quantity is $\frac{2}{3}$, which means that due to the noncooperation, the firms end up producing more and setting a lower price than the optimal monopoly case.

- Consider another property of Cournot. Let $C_i(x_i) = c_i x_i$. Denote:

$$\hat{c} = \sum_{i=1}^{n} c_i s_i,$$

ie, we have the share weighted average marginal cost. Then:

$$\frac{P - c_i}{P} = \frac{s_i}{\epsilon} \Rightarrow \frac{Ps_i - c_i s_i}{P} = \frac{s_i^2}{\epsilon} \Rightarrow \frac{\sum_i Ps_i - \sum_i c_i s_i}{P} = \frac{\sum_i s_i^2}{\epsilon} \Rightarrow$$

$$\frac{P - \hat{c}}{P} = \frac{H}{\epsilon},$$

where H is the Herfindahl index. So a higher H (more concentration) leads to higher markups. A lower H means the market is more fragmented, and this leads to lower markups.

- Note that for $c_i = 0$ and $A = B = 1$, $lim_{n\to\infty} X^* \to 1$, ie the competitive outcome. Is this a general result for Cournot games? This idea motivated a paper by Novshek and Sonenshine. Given a market with U shaped AC curves, as in G-14.6, a maximium of 10 firms can be sustained at their minimum efficient scale given that demand. Thus, the number of firms does not go to infinity. And at the same time, for a number like 10 firms, it's hard to imagine that firms would not recognize that they have some strategic ability in influencing the market price so you might think we will not reach the competitive outcome.

- So in the paper they consider a cost curve $C(q)$ such that $arg\ min\ \frac{C(q)}{q} = 1$. Let $c = C'(1)$. Then consider a family of cost structures of the form:

$$C_\alpha(q) = \alpha C(\frac{q}{\alpha}).$$

Claim: For all α,

$$arg\ min_x\ \frac{C_\alpha(x)}{x} = \alpha,$$

and at α, $C_\alpha(\alpha) = c$. So by shifting α, the cost curves shift to the left maintaining the minimum efficient scale at c. See G-14.7.

- **Theorem** Let Q^{pc} be the perfectly competitive quantity. In an Cournot NE, the equilibrium outcome is in the interval $[Q^{pc} - \alpha, Q^{pc}]$, and as $\alpha \to 0$, $Q \to Q^{pc}$.

15 Lecture 15: October 20, 2005

15.1 More on Cournot and Bertrand

Finalizing the Classical Cournot Model

- Given the Cournot model of firms competing in quantities, we have the following (nice) imperfectly competitive results:
  - (1) Get strategic behavior of firms.
  - (2) As $n \rightarrow \infty$, $Q \rightarrow Q^{pc}$, so this is like an intermediate structure between monopoly and perfect competition.
  - (3) Markups are directly related to elasticities.
  - (4) Markups are directly related to market shares.
  - (5) Market shares are directly related to costs.
  - (6) And as a result of (5), less efficient firms remain in the market with a lower market share.
- Recall in the monopoly model, we could make either P or Q our choice variable and get the same result. In the oligopoly setting, switching from quantity (Cournot) to price (Bertrand) has dramatically different results.

5. Bertrand Games

- Consider again n firms, $i = 1 \ldots n$ which all produce a homogenous good.
- The strategy space for all firms is a price $p_i \in \Re_+$.
- Market demand is denoted $Q(p)$ with $Q'(p) < 0$, continuous.
- We can also denote firm specific demand as:

$$q_i(p_i, p_{-i}) = \begin{cases} 0, & if\ \exists\ j \ni p_j < p_i \\ \dfrac{Q(p)}{m}, & if\ p_i \leq p,\ p = min_j\ (p_j),\ m = \#\{j | p_j = p\} \end{cases}$$

So the firm faces demand of zero if they aren't (one of) the lowest price firm(s) due to the usual Bertrand argument. Also, I have assumed if the firm is one of the lowest price firms, all lowest price firms share the market equally. This is NOT required. It could be that for $p_i \leq p$ defined above, $q_i = \tilde{q}_j$ (possibly different for all low price firms) with all low price firms producing postive quantities and $\sum_j \tilde{q}_j = Q(p)$, ie they satisfy the whole market.

- Firm payoffs are thus:

$$\pi_i(p_i, p_{-i}) = q_i(p_i, p_{-i})(p_i - c_i),$$

where this might be zero if $q_i = 0$ (ie, when firm i is not the lowest price firm). So we have assumed CONSTANT marginal costs here. This is pretty strong, but it turns out that we need this assumption to get existance in pure strategies.

- Let $c^{(2)} = c_i$ such that there exists 1 index j where $c_j < c_i$ or $c_j = c_i$. So $c^{(2)}$ is just the second lowest costs of all the firms (but it could be the lowest overall if two firms have the same low cost).

- **Proposition** The "unique" NE of the Bertrand pricing game has $p = c^{(2)}$. Furthermore, if $c_i < c^{(2)}$, $q_i = Q(p)$, ie if firm i has a lower cost than any other firm, it wins the entire market, but still sets a price of $p = c^{(2)}$. Note that we need $c^{(2)} < p^m$ for this to hold. This just means that if the second lowest cost firm has a cost above the monopoly price, the lowest cost firm will still set the monopoly price (and not $c^{(2)}$), because the monopoly price is optimal.

- **Corollary** If $c_i = c \; \forall \; i$, then price = MC and the market is shared.

- **Corollary** If $c_1 < c^{(2)}$, $p = c^{(2)}$ and firm 1 serves the whole market, ie $\tilde{q}_1 = Q(p)$.

- So if one firm has a lower cost than everyone else, we don't get a monopoly solution, but instead price is just a bit above the competitive price at $p = c^{(2)}$.

- **Corollary** If there are two firms and both have the lowest cost in the market, these two firms share the market and price is driven down to their common marginal cost. So we only need two firms in the market with a lowest cost to eliminate the wedge between price and marginal cost.

- So two firms are enough to get perfect competition in the Bertrand world. This was not the case in Cournot. The difference is that price is a far more competitive variable than quantity. Consider G-15.1. The 2 firm Cournot model has each firm facing the residual demand once the other firm has choosen its quantity. This yields the usual Cournot result and a small deviation in quantity produced has relatively small consequences for revenues. Now consider G-15.2. This is the 2 firm Bertrand model where firms share the market equally if they set the same price. The residual demand for firm 1 is now equal to zero for prices above p_2, it equals $Q(p)/2$ when they set the same price, and then it equals the whole market when firm 1 sets a lower price than firm 2. At $p_1 = p_2$, demand is INFINITELY elastic. Thus, a price change from this point results in an enormous change in quantity. This drives down the price to the competitive level because firms will always want to undercut. Price change in Bertrand thus have a much larger effect on both firm's payoffs than quantity does in a Cournot world.

- **Remark** So what's the problem here? Clearly our assumptions of homogenous goods and no capacity constraints seem a bit unrealistic and both have a large impact on the differing results of Cournot and Bertrand. However, Bertrand seems more attractive because it uses price as the control variable.

- **Remark** Also, note that Bertrand only yields (close to) the competitive price if marginal costs are constant. Consider G-15.3 where we have increasing marginal costs for firms 1 and 2. Given the residual demand for firm 1, it will want to raise it's price a bit and restrict output. Thus the perfectly competitive outcome does not arise. We lose the existance of a pure strategy NE when marginal costs are not constant.

6. Differentiated Products Bertrand

- See Xavier Vives (JET 1986) for a more general formulation of this with hetergenous firms. Here we present a special case of that paper.
- Consider 2 firms that produce differentiated, substitutable goods.
- Costs are $C_i(Q_i) = cQ_i$, for $i = 1, 2$.
- Demand for firm i's product, $Q^i(p_i, p_j)$, satisfies Assumptions A1 (subscripts represent partials):
  - (1) $Q^i_1(p_i, p_j) < 0$, downward sloping in your own price.
  - (2) $Q^i_2(p_i, p_j) > 0$, substitutes.
  - (3) $Q^i_{1,2}(p_i, p_j) > 0$, slope becomes less negative as $p_j \uparrow$.
  - (4) $2Q^i_1(p_i, p_j) + Q^i_{1,1}(p_i, p_j)(p_i - c) < 0$, profit function is concave.
- Thus the objective function for firm i:

  $$\pi^i(p_i, p_j) = Q^i(p_i, p_j)(p_i - c).$$

  FOC:

  $$Q^i(p_i(p_j), p_j) + Q^i_1(p_i(p_j), p_j)(p_i(p_j) - c) = 0.$$

  And by the concavity of the profit function (A1.4 above),

  $$\exists!\ \hat{p}_i(p_j),$$

  that solves.
- More next time.

16 Lecture 16: October 25, 2005

16.1 More on Differentiated Products Bertrand

- Recall **Assumptions A1** from last time involving 2 firms producing a differentiated product (substitutable):
  - (1) $Q_1^1(p_1, p_2) < 0$, downward sloping in your own price.
  - (2) $Q_2^1(p_1, p_2) > 0$, substitutes.
  - (3) $Q_{1,2}^1(p_1, p_2) > 0$, slope becomes less negative as $p_2 \uparrow$.
  - (4) $2Q_1^1(p_1, p_1) + Q_{1,1}^1(p_1, p_2)(p_1 - c) < 0$, profit function is concave.
- The profit function for firm 1 can be written:

$$\pi^1(p_1, p_2) = Q^1(p_1, p_2)(p_1 - c_1).$$

FOC:

$$0 = \pi_1^1(p_1(p_2), p_2),$$

or,

$$0 = Q^1(p_1, p_2) + Q_1^1(p_1, p_2)(p_1 - c_1).$$

- Totally differentiate:

$$\pi_{11}^1 p_1'(p_2) + \pi_{12}^1 = 0,$$

or,

$$p_1'(p_2) = -\frac{\pi_{12}^1}{\pi_{11}^1} \geq 0.$$

The numerator is positive because of A1.4 and the denominator is negative by the SOC. Thus the response functions for both firms 1 and 2 are upward sloping.

- Because $\pi_{12} > 0$, we have strategic complements which is the opposite of Cournot.
- None of what we have done so far has given us existence or uniqueness. We need some further assumptions.
- **Assumptions A2** Suppose $\exists\ \underline{p}_i > c_i$ s.t.

$$Q^i(\underline{p}_i, c_j) + Q_i^i(\underline{p}_i, c_j)(\underline{p}_i - c_i) = 0.$$

So this means even if firm j is setting price equal to marginal cost, firm i can still find a price above its marginal cost that satisfies its FOC. Thus a firm CANNOT drive its competitor out of the market by lowering its price to marginal cost. See G-16.1. So even with this assumption, we don't necessarily have a solution. We need more.

- **Assumptions A3** (Seade's Condition). Assume:

$$\left|\frac{\partial^2 \pi^i}{\partial p_i^2}\right| \geq \sum_{j \neq i} \left|\frac{\partial^2 \pi^i}{\partial p_i \partial p_j}\right|.$$

  This is just Friedman's condition from last time which gave us the uniqueness (and stability) of the Cournot solution.

- If $n = 2$, this implies:

$$p_2'(p_1) < 1,$$

  and,

$$p_1'(p_2) < 1 \Longrightarrow \frac{1}{p_1'(p_2)} > 1.$$

  See G-16.2. So Seade's gives us existence and uniqueness !!

- All along, we have been assuming firms choose quantities or prices simultaneously. Really, we are just saying that firms must choose before knowing the choices of their competitors. So this is an ok assumption. With both Cournot and Bertrand, the strategy spaces are real numbers. With a stackelberg setup, where one firm leads and one follows, the strategy space is now a set of (best response) functions.

16.2 Dynamic Bertrand (Stackelberg in Prices)

- Consider a homogenous good produced with constant marginal cost, c. Demand is:

$$P(Q) = a - Q, \quad Q(P) = a - P.$$

- Suppose firm 1 is the leader and 2 is the follower. Firm 1 chooses a price and firm 2 must react. Thus the strategy of firm 1 is to choose $p_1 \in \Re_+$ and the strategy of firm 2 is to choose a mapping, $f : \Re_+ \mapsto \Re_+$. A subgame is ANY choice of p_1 by firm 1. Subgame perfection requires the function $\hat{p}_2(p_1)$ satisfies:

$$\hat{p}_2(p_1) \in arg\ max\ \pi^2(p_1, p_2)\ \forall\ p_1.$$

- We have the similar problem of continuous prices leading us to no equilibrium. (Ie, if firm 1 sets a price strictly above c, firm 2 wants to undercut, but by how much (how little)?)

- One solution is to discretize the state space. Suppose prices are discrete so the strategy space for firm 1 is now:

$$S_1 = \{c, c + \epsilon, c + 2\epsilon, \ldots, c + n\epsilon, \ldots\},$$

  where ϵ might be a penny.

- If $c + \epsilon < p_1 < p^m$, the optimal response of firm 2 is:

$$\hat{p}_2(p_1) = p_1 - \epsilon.$$

  Firm 2 undercuts and gains the whole market.

- If $p_1 = c + \epsilon$, the optimal response of firm 2 is:

$$\hat{p}_2(p_1) = p_1.$$

  Firm 2 shares the market with firm 1. Undercutting would gain him the whole market but with zero profits.

- If $p_1 = c$, the optimal response of firm 2 is:

$$\hat{p}_2(p_1) = c \ \text{ OR } \ \hat{p}_2(p_1) > c.$$

  Either of these strategies yield profit of zero to both firms.

- Clearly, firm 1 has an incentive to choose $p_1 = c + \epsilon$ since it yields him positive profits. Firm 2 responds by setting the same price. So the subgame perfect Nash is:

$$(\hat{p}_1, \hat{p}_2(p_1)) = (c + \epsilon, c + \epsilon).$$

  As $\epsilon \to 0$, the equilibrium tends to (c, c), the static Bertrand solution!

- So adding a dynamic (stackelberg) nature to our Bertrand game really hasn't helped the situation. Next we will turn to dynamic Cournot.

17 Lecture 17: October 27, 2005

17.1 More on Cournot versus Bertrand

Dynamic Cournot (Stackelberg in Quantities)

- Consider a market setup where firm 1 moves first and chooses a quantity and then firm 2, observing q_1, choose its quantity.
- Demand is:
$$P(Q) = a - bQ = a - bq_1 - bq_2.$$
- Costs: constant and symmetric marginal cost so:
$$C_i(q) = cq.$$
- Recall from 602, the monopoly quantity, price, and total output are:
$$q^m = \frac{a-c}{2b}, \quad p^m = \frac{a+c}{2}, \quad Q^m = \frac{a-c}{2b}.$$
Similarly for simultaneous cournot:
$$q^c = \frac{a-c}{3b}, \quad p^c = \frac{a+c}{3b}, \quad Q^c = \frac{2(a-c)}{3b}.$$
- This setup is useful for considering entry deterence models.
- Consider the scenario that firm 2 threatens to set the monopoly quantity in period 2, then firm 1 should set $q_1 = 0$. But this wouldn't be sequentially rational and hence not a subgame perfect NE.
- A subgame is a choice of q_1. Firm 2's payoff is:
$$\pi^2(q_2; q_1) = (a - bq_1 - bq_2 - c)q_2.$$
FOC induces:
$$\hat{q}_2(q_1) = \frac{a - c - bq_1}{2b}.$$
This is the best response function and it, in fact, is exactly the same as the best response function (correpondence) in the simultaneous Cournot game. The difference is for firm 1.
- Firm 1 optimizes:
$$\pi^1(q_1, q_2) = (a - bq_1 - bq_2(q_1) - c)q_1.$$
$$\pi^1(q_1, q_2) = (a - bq_1 - b[\frac{a - c - bq_1}{2b}] - c)q_1.$$
We can then take the FOC as usual.

- See G-17.1. Here we graph the best reponse function for firm 2 (the same in both simultaneous and dynamic Cournot) but also include the best response function for firm 1 of the simultaneous Cournot game. Consider fixing $\overline{q}_2$. Then the best that firm 1 can do is set $q_1 = q_1(\overline{q}_2)$. Thus firm 1's profit function must be maximized there as shown. Indeed for any choice of q_2, by definition, firm 1 must maximize profits on his best reponse function. So at the Cournot quantity, point E, clearly the profit function is NOT tangent to firm 2's best response function which is required for the Dynamic Cournot game. Thus the dynamic Cournot solution must be somewhere like point F. Given the dashed line where total quantity produced is equal to the simultaneous Cournot quantity, the dynamic Cournot TOTAL quantity is higher! Note:

$$q_1^c(Dynamic) > q_1^c(Simultaneous).$$

$$q_2^c(Dynamic) < q_2^c(Simultaneous).$$

$$Q^c(Dynamic) > Q^c(Simultaneous).$$

- Still, this doesn't help us reconcile the Cournot and Bertrand models. In Vives - JET - 1985, he shows the even with differentiated products, price is always a much more competitive variable than quantities so:

$$Q^M < Q^C < Q^B < Q^{PC},$$

and as $n \to \infty$, this ordering is maintained though the Cournot and Bertrand quantities converge to the perfectly competitive quantity.

Kreps - Schenckman: Cournot and Bertrand Together

- So consider a combination of both models. How about an entrepreneur that is considering building a hotel. He first chooses the size of his hotel, and then faced with a fixed capacity of rooms, he competes in prices with other hotels. So this is like having two stages: compete first in quantities (Cournot) and then in stage two, we have Bertrand price competition. Suppose both stages are simultaneous (ie, both firms first choose quantities and then they choose prices).
- Do we get the best of both worlds in this case ? It seems pretty good, but there are a couple problems. First, see G-17.2. With fixed capacity, the marginal cost curve is constant for quantities up to the capacity limit and then goes vertical. This is a case of convex costs and as we have said, we may not get a pure strategy NE in stage 2 in the Bertrand game. The solution might exist in mixed strategies but this is not attractive. But maybe we can live with it.
- The second problem is illustrated in G-17.3. In normal Bertrand competition, the firm with the lower price steals the entire market. But what if he cannot supply the entire market due to capacity constraints? Firm 2, with a higher price, may still get some market share. How do we allocate customers to each firm ? Two methods:

– Efficient Rationing Rule. Consumers who value the product the most get the low price and the higher price firm effectively faces a residual demand curve that is simply a horizontal-shift-inward-version of the original demand.

– Proportional Rationing Rule. Suppose all consumers get a fixed chance, say $\frac{\overline{q}_1}{Q(p_1)}$ of attaining the product at the low price (where $\overline{q}_1$ is firm one's capacity and $Q(p_1)$ is the total market demand at p_1). This means that firm 2 effectively faces a residual demand curve which is a pivoted-in-version of the original demand.

18 Lecture 18: November 1, 2005

18.1 Horizontal Mergers

- We next consider contractual agreements to change the structure of the market. Ie, if one firm buys another's productive assets. An example of this was the recent purchase of AT&T by SBC.
- There are other classes of mergers including:
  - Vertical Mergers. An upstream firm buys a downstream firm for example. GE once attempted to buy Honeywell but the merger was blocked by the European Commission.
  - Conglomerate Mergers. Unrelated firms merge their product lines.
- There are clear antitrust issues for horizontal mergers though not quite as much for the other two types. Three important US acts deal with what is an acceptable restraint to trade: The Clayton, Sherman and the FTC act.
- The Hart-Scott-Rodino Act of 1986 states that any merger that has a value of at least 50 million dollars (ie, the target firm is worth at least 50 million) must be reported to the FTC or the DoJ. The DoJ for example can then decide to challenge the merger and the courts then decide to stop it or not. In Europe, the European Antitrust Commission actually decides whether or not to allow the merger. In the US, if the DoJ decides to investigate further, in most cases the firms withdraw in order to avoid the costly litigation that would ensue.
- If the merger is investigated in the US, courts refer to the "Horizontal Merger Guidelines" publication. There are 5 major sections.
  - (1) Does the merger have a significant impact on industry concentration (HHI).
  - (2) Does the change in the concentration of the industry lead to an increase in prices for consumers? Are there other competitive effects?
  - (3) Does the merger effect entry conditions.
  - (4) Are there efficiency reasons to justify the merger?
  - (5) Would exit by one of the firms occur without the merger? Ie, if one firm is failing and being bought up, blocking the merger is fruitless because once the failing firm left, only one firm would be left anyway.

  We consider these sections in a bit more detail.

Merger Guidelines 1: The Effects on Concentration

- Suppose there are n firms in the industry. Officials might calculate:

$$s_i = \frac{x_i}{X},$$

where x_i might be firm's sales, revenues etc and X is the total for the whole industry. We could then calculate:

$$HHI = \sum_{i=1}^{n} s_i^2 \ \in [0, 10000].$$

- The problem with this is determining n. How big is the market? What defines the market? What are its boundries? What is the product?
- For homogenous goods, this might be easier, but when products are differentiated, we might consider cross price elasticities, but these are hard to calculate and the cut-off is still going to be arbitrary.
- We consider the "Hypothetical Monopolist Experiment." Start with a narrow market definition and consider all firms that fall in that narrow market as one firm. Then ask, can that monopoly profitably engage in a Small and Significant Nontransitory Increase in Price (SSNIP) of say 5 percent? For a narrow definition, it will probably not be possible. So extend the market and consider the larger monopolist and see if they could do a SSNIP. Repeat this until we have the smallest possible market definition such that a SSNIP is possible. We might do this with surveys or natural experiments might present themselves from time to time. Once we have our market, we can consider measures of market concentration.
- Given our market the DoJ considers the following concentration guidelines:
  - If $HHI \in (0, 1000)$, no concern.
  - If $HHI \in (1000, 1800)$, moderate concern and investigate if the HHI would increase by 100 points with the merger.
  - If $HHI \in (1800, 10000)$, extreme concern and investigate if the HHI would increase by 50 points with the merger.

Merger Guidelines 2: Competitive Effects

- There are two types of competitive effects we might be concerned with:
  - (1) Coordination Effects: Is it easier to collude after the merger?
  - (2) Unilateral Effects: Will firms have more power over price following the merger?

Merger Guidelines 4: Efficiency Effects

- This a pretty vague and difficult to quantify.
- One view might be that firms will save SO much by merging that in fact prices will FALL following the merger so everyone is better off. But this is hard to sell.

- Another view is that though firms will have power over price after the merger, the profits of the merged firm will "outweigh" the loss in consumer surplus from the higher prices. Again, the antitrust official is probably going to put more weight on the consumer's plight so this is also a hard sell.

19 Lecture 19: November 3, 2005

19.1 Theory of Horizontal Mergers

Salant, Switzer, Reynolds

- Consider a Cournot model with n firms and linear demand:

$$P = b - Q.$$

- Costs:

$$C_i(q) = cq.$$

- Which induce payoffs to each firm:

$$\pi_i(n) = \left(\frac{b-c}{n+1}\right)^2.$$

- Now suppose $m+1 \leq n$ firms decide to merge leaving $n-m-1$ firms in the fringe and a total of $n-m$ firms including the newly merged firm. Profits per firm (including the merged firm) are:

$$\pi_i^{post}(n-m) = \left(\frac{b-c}{n-m+1}\right)^2.$$

- Total profits among the $m+1$ firms prior to merging:

$$\pi_i^{pre}(n) = (m+1)\left(\frac{b-c}{n+1}\right)^2.$$

- So comparing these last two equations (the profits of the $m+1$ firms pre and post merger) should tell us when the merger is profitable. Consider:

$$
\begin{aligned}
\Delta &= \pi_i^{post}(n-m) - \pi_i^{pre}(n) \\
&= \left(\frac{b-c}{n-m+1}\right)^2 - (m+1)\left(\frac{b-c}{n+1}\right)^2 \\
&= \frac{1}{(n-m+1)^2} - \frac{m+1}{(n+1)^2} \\
&= \frac{1}{(n(1-m/n+1/n))^2} - \frac{n(m/n+1/n)}{(n+1)^2} \\
&= \frac{1}{n^2(1-m/n+1/n)^2} - \frac{n(m/n+1/n)}{(n+1)^2} \\
& \text{let } r = \frac{m}{n} \\
&= \frac{1}{n^2(1-r+1/n)^2} - \frac{n(r+1/n)}{(n+1)^2}
\end{aligned}
$$

[I believe there is an error above regarding the $(b-c)^2$ terms canceling]. Note if $m = 0$, $\Delta = 0$. Salant, et. al. show that Δ plotted as a function of r looks like G-19.1. It first is negative, crosses into positive territory at say, m^*, and then increase up to the point where $r = 1$ or $m/n = 1$ or all firms merge into a monopolist.

- Thus there exists a large enough m such that merging is profitable. But how big does m have to be (ie, how big of a merged company do we need to create)? The authors find $m^* \approx 0.8$!! So we need a lot of firms to merge.

- Note as $n \uparrow$, $m^* \uparrow$. Why are some mergers unprofitable in this setup? The problem with the model is we assume symmetric costs and constant returns to scale. Thus, when a merger occurs, the output is still divided equally among all the firms still in the industry. Thus fringe firms benefit from the merger (higher prices). This is a (huge) externality. Thus we might not take Salant, et. al. very seriously due to this unlikely result.

- Next we relax the CRS assumption by introducing a decreasing returns to scale production function.

McAfee and Williams

Perry and Porter

- Consider a Cobb-Douglas production function:

$$f(l,k) = l^{1/2}k^{1/2}.$$

- Assume firm i has capital stock, k_i, which is fixed. Labor is variable and the wage rate is normalized to 1. Thus,
$$f(l_i; k_i) = l_i^{1/2} k_i^{1/2}.$$
- Costs:
$$C(q_i; k_i) = \frac{q_i^2}{2k_i},$$
induces marginal costs, $mc(q_i; k_i) = \frac{q_i}{k_i}$. So these vary firm by firm depending on their capital stock. See G-19.2.
- Demand:
$$P = 1 - Q.$$
- Assume we are again in a Cournot world and denote $\beta_i = \frac{k_i}{1+k_i}$ which proxies the size of the firm. Also,
$$B = \sum_{i=1}^{n} \beta_i.$$
- In equilibrium, the authors show:
  - (1) $Q = \frac{B}{B+1}$.
  - (2) $p = \frac{1}{B+1}$.
  - (3) $q_i = \frac{\beta_i}{B+1}$.
  - (4) $s_i = \frac{\beta_i}{B+1}$.
  - (5) $B = \frac{1}{elasticity}$.
  - (6) Welfare, $W = CS + \sum_i \pi_i = \frac{1}{2}(1 + H\frac{B}{1+B})\frac{B}{1+B}$, where H is the Herfindahl index.
- So for a given Q, a social planner (SP) would simply allocate the production to those plants with the highest productivity and he would equalize the marginal cost. If all firms had the same level of k_i, then all firms would be allocated the same proportion of the output. However, now k_i varies firm by firm. Thus,
$$q_i = \frac{\beta_i}{B+1} = \frac{k_i/(1+k_i)}{B+1} \nrightarrow mc_i(q_i) = mc_j(q_j).$$
So we do NOT get the SP outcome because each firm has a different marginal cost.
- Now suppose that two firms merge. The merged firms can still produce the sum of their old outputs at their old costs, but they probably can do even better. The merged

firm will instead equalize the marginal costs of production between their plants. This means there is a social gain! The merged firm will produce more efficiently. Also, post merger, the market is more concentrated so $H \uparrow$. But this leads to a welfare GAIN via result (6)! So the cost saving technology (Decreasing Returns to Scale) makes the merger welfare improving.

- So are mergers really a bad thing ? More next time.

20 Lecture 20: November 8, 2005

20.1 More on Horizontal Mergers

More on McAfee and Williams

- How much of their results are a consequence of the functional forms they assume? Possibly a lot.
- Consider a market with n firms and two of them merge. The new capital of the merged firms are $k_1 + k_2$. Let β^m be equal to $\frac{k_1 + k_2}{k_1 + k_2 + 1}$, again be our proxy for the size of the merged firm. Mathematically, it is easy to see that:

$$Max\{\frac{k_1}{1+k_1}, \frac{k_2}{1+k_2}\} < \beta^m < \beta_1 + \beta_2.$$

  So the merged firm has a size that is larger than the larger of the two firms individually, but not as big as just adding the individual sizes together. Thus $B^m = \sum \beta_i^m < B = \sum \beta_i$. Thus the overall market size falls. So $Q^m < Q$, total output falls and prices must rise.
- What about welfare? Well, W is increasing in H (which is rising with the merger) and is also increasing in B (which is falling with the merger). Thus welfare consequences are ambiguous.
- See G-20.1. We plot the case of an $n = 3$ industry and firms 1 and 2 consider merging. We have a region where the merger is profitable, where a merger is welfare improving, and a region where the H index will be above 1800 and the DoJ will step in and stop things. As a result, we see there is both the possibility that a welfare improving merger will be stopped (type I error) and also the possibility that a welfare reducing merger will be allowed (type II error).

Farrell and Shapiro

- They argue that firms merge due to cost synergies so considering the Herfindahl index as a criteria for stopping mergers is incorrect. A high H could actually be a good (welfare improving) result.
- In general, there are three sources of inputs into a total welfare function:

$$W = \Pi^I + \Pi^O + CS,$$

  where we have profits of firms inside the merger, profits of those on the fringe, and consumer surplus. Thus,

$$dW - d\Pi^I = d\Pi^O + dCS.$$

  Getting a handle on $d\Pi^I$ will be tough, but we do know it is POSITIVE (revealed preference).

- Thus,

$$dW \geq d\Pi^O + dCS.$$

So if we can find conditions for the RHS to be positive, we know the merger must be welfare improving.

- The authors find that if the number of merging parties is less than the number of non-merging parties, then the merger will be welfare improving. This is NOT inconsistent with former works because, say in S/S, only mergers with an $m^* \geq 0.8$ would even be seen by the DoJ so it makes sense that we are restricting $d\Pi_I > 0$.

- **Lemma** Suppose the Cournot game satisfies the stability property (Friedman's Condition). Then if firm 1's output rises exogenously, total output in the industry also rises. See G-20.2. The dotted line shows level sets of equal total output. The best response functions are steeper and shallower than this line and an exogenous increase in Q_1 shifts firm 1's BR curve to the right. Total output must rise.

- **Proposition** Assume MR is decreasing. Then price rises post merger (CS falls) iff:

$$p(\overline{X}) - c_x^m < p(\overline{X}) - c_x^1 + p(\overline{X}) - c_x^2.$$

So if the premerger markups over marginal cost for two firms sum to something greater than the margin of price over marginal cost where c_x^m is the marginal cost premerger at output $\overline{x}_1 + \overline{x}_2$, then if the firms did merge, they would cut output and raise the price.

- So mergers are welfare improving ONLY IF there are LARGE cost synergies.

20.2 Dynamic Oligopoly

- Now we study situations where the incentives of a firm include future competition with other firms.

- We will model this in two stages. First a firm makes a choice about some strategic variable (investment in a new product, R and D expenditure, the decision to bundle, capacity choices, etc). Then in stage 2, firms will compete. The stage one decision somehow affects the market structure of stage 2.

- Start with 2 firms with strategies, s_1 and s_2, that are choosen simultaneously and yield payoffs:

$$\pi^i(s_1, s_2), \quad i = 1, 2.$$

- FOCs imply:

$$\pi_1^1(s_1(s_2), s_2) = 0.$$

$$\pi_2^2(s_1, s_2(s_1)) = 0.$$

- Differentiate again:

$$\pi_{11} s_1'(s_2) + \pi_{12}^1 = 0,$$

or,

$$s_1'(s_2) = -\frac{\pi^1_{12}}{\pi^1_{11}},$$

and,

$$s_2'(s_1) = -\frac{\pi^2_{12}}{\pi^2_{22}}.$$

- To get stability, we need:

$$\left|\frac{1}{s_1'(s_2)}\right| > \left|s_2'(s_1)\right|.$$

  See G-20.3.

- So clearly, $\pi^i_{ii} < 0$ for $i = 1, 2$, but other terms depend on if the products are strategic complements or strategic substitutes. For either case, it can be shown (in hand written notes),

$$\pi^1_{11}\pi^2_{22} - \pi^1_{12}\pi^2_{12} \geq 0,$$

  is the sufficient condition for stability.

- So consider a dynamic 2 period game where in stage 1, firm 1 (say) selects $K \in \Re$ where K is a strategic variable like above. Then in stage 2, firms 1 and 2 observe K and then play the "canonical game." Assume:

$$\pi^1(s_1, s_2, K) \ \ and \ \ \pi^2(s_1, s_2).$$

  Note that even though we don't write firm 2's profit function as directly depending on K, it does depend indirectly on K through the strategies.

- We will seek a Subgame Perfect NE.

21 Lecture 21: November 10, 2005

21.1 More on Dynamic Oligopoly

- Recall we are considering a 2 period game where in period 1, firm 1 chooses K and in period 2, both firms observe K and play an (s_1, s_2) simultaneous move game with payoffs, $\pi^1(s_1, s_2, K)$ and $\pi^2(s_1, s_2)$.

- We make some assumptions:

  - (A1) $sign\left(\frac{\partial \pi^1}{\partial s_2}\right) = sign\left(\frac{\partial \pi^2}{\partial s_1}\right)$. So the cross effects are in the same direction but not necessarily the same magnitude.
  - (A2) $sign\ (\pi^1_{12}) = sign\ (\pi^2_{12})$. So we have either both strategic substitutes or both strategic complements.
  - (A3) $\forall\ K, s_1,\ \exists\ \overline{s}_2 \ni\ \pi^2(s_1, \overline{s}_2) = 0$. So this is a free exit property.
  - (A4) $\forall\ K, \exists!$ NE to the second stage game.

- So we consider two types of games: Deterred entry games and Accommodated entry games. Firm 1 can either set K such that firm 2 decides not to enter or firm 1 can set K such that firm 2 still enters.

- **Deterred Entry** How can choices of K deter entry? Let $(s_1(K), s_2(K))$ be the equilibrium which dictates play in the second period. Firm 2 will exit iff:

  $$\pi^2(s_1(K), s_2(K)) \leq 0.$$

  So how does K impact the profits of firm 2? Not directly, but indirectly through the firm's strategies. Consider:

  $$\frac{d\pi^2}{dK} = \pi^2_1 \frac{ds_1}{dK} + \underbrace{\pi^2_2}_{=0\ by\ env} \frac{ds_2}{ds_1}\frac{ds_1}{dK} = \pi^2_1 \frac{ds_1}{dK}.$$

  So what is the sign of the whole thing? If this term is negative, we call the leader "Tough." K might be market stealing advertising for example. If the term is positive, we call the leader "Soft." K might be market expanding advertising.

- **Accommodated Entry** In a SPE, firm 1 gets $\pi^1(s_1(K), s_2(K); K)$. The FOC is:

  $$0 = \pi^1_1 \frac{ds_1}{dK} + \pi^1_2 \frac{ds_2}{dK} + \pi^1_K = \underbrace{\pi^1_2 \frac{ds_2}{dK}}_{SE} + \underbrace{\pi^1_K}_{SAE},$$

  because again $\pi^1_1 = 0$ by the envelope condition. So the second term we have left is the Stand Alone Effect (SAE) which is illustrated in G-21.1. If firm 2 was not in the market, firm 1 would choose $\hat{K}$. The first term is the Strategic Effect (SE) which could be negative or positive. If $SE > 0$, $K^* > \hat{K}$ and if $SE < 0$, $K^* < \hat{K}$, as shown in the graph.

- Now we want to know how K affects the firm's profits. It is going to depend on if (1) the leader is tough or soft and (2) if the products that are being produced are strategic substitutes or complements. Note that if K enters the profit function additively, then $SE = 0$.

- So consider again the strategic effect:

$$SE = \pi_2^1 \frac{ds_2}{dK} = \pi_2^1 \frac{ds_2}{ds_1} \frac{ds_1}{dK} \neq 0$$

in general. If this is positive, the leader might want to increase K and if negative, it might want to decrease K relative to the Stand Alone Effect.

- Consider the FOCs of the Nash game:

$$\pi_1^1(s_1, s_2; K) = 0,$$

$$\pi_2^2(s_1, s_2) = 0.$$

Totally differentiate:

$$\pi_{11}^1 ds_1 + \pi_{12}^1 ds_2 + \pi_{1K}^1 dK = 0,$$

$$\pi_{21}^2 ds_1 + \pi_{22}^2 ds_2 = 0.$$

Solve using Cramer's rule:

$$\frac{ds_2}{dK} = \frac{\pi_{1K}^1 \pi_{12}^2}{\pi_{11}^1 \pi_{22}^2 - \pi_{12}^1 \pi_{12}^2}.$$

The denominator is positive by invoking the Stability Condition !! The term π_{12}^2 depends on strategic substitutes or complements and π_{1K}^1 is a problem. What we do know is:

$$sign\ \pi_{1K}^1 = sign\ \frac{ds_1}{dK}.$$

So,

$$sign\ \frac{ds_2}{dK} = sign\ \frac{ds_1}{dK} * \ sign\ slope(RC).$$

So,

$$sign(SE) = sign(\pi_2^1 \frac{ds_1}{dK} slope(RC)) = \underbrace{sign(\pi_1^2 \frac{ds_1}{dK})}_{soft/tough} * \underbrace{sign\ Slope(RC)}_{strat\ subs/comps}.$$

- So overall we have a matrix of effects which determine the sign of the Strategic Effect:

	Soft	Tough
Strategic Substitutes	$SE < 0$ Lean/Hungry	$SE > 0$ Top Dog
Strategic Complements	$SE > 0$ Fat Cat (Acc), Lean/Hungry (Deter)	$SE < 0$ Pussy Cat (Acc), Top Dog (Deter)

- See Vincent's notes for examples of each.

22 Lecture 22: November 15, 2005

22.1 More on Dynamic Oligopoly

- Recall our table from last time depicting the type of strategic effect we have in our market. Which box we are in will have implications for policy. Ie, if we think we have Cournot competitors, but actually firms compete in price, the policy you instate may have exactly the opposite of the desired impact. So when we have imperfectly competitive markets, be cautious about policy recommendations.

Winston - Tying

- We consider tying in a strategic context.
- One view of tying is as a damaging economic device. For it to work, you need:
  - (1) A monopoly in one industry.
  - (2) Two distinct products that are being tied (ie, you can only buy A if you also buy B).
  - (3) Commerical effects must exist.
- The Chicago school responded to this view by saying that tying was, in fact, not harmful because there is only ONE monopoly rent. The net value of the monopolized product, say A, to the consumer is zero. So to force the consumer to also buy the tied in product, say B, the monopolist has to lower its price on A. This exactly offsets the gains from selling B, so it is impossible to leverage your monopoly into another market.
- So consider a model of 2 firms, 1 and 2, with firm 1 producing A (as a monopolist) and B. Firm 2 only produces B.
- Consumers value one unit of A at γ. They have inelastic demand for one unit of good B generated by a differentated Bertrand setting. In particular, firm 1 faces demand:

$$x^1(P_{B1}, P_{B2}),$$

  where P_{Bi}, $i = 1, 2$ are the prices firm 1 and 2 charge for B. Assume $x^1_1 < 0$, $x^1_2 > 0$, and $x^1_{12} > 0$.
- Firm 1 has FIVE options regarding his pricing strategy:
  - (S1) No bundling: sell goods independently: (P_A, P_{B1}).
  - (S2) Pure bundling: sell only bundle: (P_{bundle}).
  - (S3) Mixed bundling: sell both bundle and A and B separately: $(P_A, P_{B1}, P_{bundle})$.
  - (S4) Alternative: bundle and A alone: (P_A, P_{bundle}).
  - (S5) Alternative: bundle and B alone: (P_{B1}, P_{bundle}).

- In reality, only S1 and S2 are relevant since S1 $\equiv$ S3 $\equiv$ S4. Also S2 $\equiv$ S5.
- If firm 1 cannot commit to sell only the bundle, we have a problem of committment. Thus Winston assumes that firm 1 makes a "design decision" that bundles A and B together tightly. So tight that it is impossible to separate them. Think MSFT and IE.
- So somehow, (see Vincent's notes), using the FOCs under S1 and S2, we see that if firm 1 offers ONLY the bundle, he will want to set the implicit price of B below what would otherwise be optimal under S1. Thus firm 2 may NOT enter the game (competition in product B). So, (I think), this counters the Chicago idea and says that bundling is worrysome from a competition perspective as the possibility to bundle is welfare damaging.

23 Lecture 23: November 17, 2005

23.1 Predation and Entry Deterence

- Consider an airline route that has only one airline that is flying between the two cities. Does that airline have power over price? Should we worry? Not if there is another airline that could easily set up shop in that market. If the firm that was flying alone in the market even tried to increase its price, entry would ensue so they are forced to keep prices down. Thus the number of firms in an industry may not be a good indicator of power over price.

- However, there may be either fixed costs (costs invariant to quantity produced) or sunk costs (fixed or variable NON-recoverable costs).

- So at what point do fixed costs become sunk?

- When is predatory pricing even feasible? There must exist some barrier to entry (maybe a cost faced by an entrant which is not faced by an incumbent).

- Gilbert said that fixed costs are barriers to entry and sunk costs are barriers to exit. Fixed costs make it hard for an entrant to enter a market (depletes its capital and makes predatory pricing easier on the part of the incumbent), while sunk costs make it more costly to cut and run.

- So consider a monopolist incumbent who has been operating for a while and faces a new entrant. They may choose to price at or below marginal (average variable) cost to drive out the new entrant. One example is the phantom ship that follows around the entrants into the shipping business, theatening to carry the load at a low price, never actually winning the contract, but keeping the price low for the entrant.

- So what are the key issues/questions?
  - (1) Would the predation strategy by feasible? Ie, can an incumbent feasibly knock out an entrant.
  - (2) Is this strategy of predation desirable/profitable?
  - (3) Is the practice socially harmful? This isn't so clear because consumers face very low prices during the predatory period.

- So we need pricing below some level of costs (average variable costs typically) and we need for the incumbent firm to have some reasonable expectation that once the entrant is driven out, he can profitable raise his price and recoup his losses.

- So why doesn't an entrant just wait it out? Clearly, there are profits to be made so why not just deal with the predation knowing that eventually the incumbent will have to raise prices. One argument for this is the Long Purse/Short Purse argument. Incumbents can hold out longer. But clearly, this also requires that the two firms have unequal access to capital markets because the entrant should be able to borrow to compete with the long purse of the incumbent.

- With perfect capital markets, the SPNE of the predation game should be NO predation!

- So what about reputational effects? If incumbent firms face a (finite) sequence of entrants, maybe they have an incentive to predate early and signal to later entrants that they are tough.

- However, according to Selten (Chain Store Paradox), if there is a last entrant, clearly the incumbent has an incentive to accommodate since there is no next stage. Working backwards, accommodating in every stage is the SPNE. So we should see NO predation in equilibrium! But we do, hence the paradox.

- Milgrom/Roberts introduced the idea of an infinitely played game with no final entrant. In this game there is one clear Nash strategy: accommodate in every period. BUT, there is also another NE strategy to deter entry (predate) in every period. The idea behind this equilibrium is that predating is only costly to the incumbent if they actually have to act on it! Thus, if the incumbent can commit to always fighting an entrant, no one will enter and the incumbent cleans up.

- Still, Milgrom and Roberts results implies that either no one enters, or firms enter and incumbents accommodate. Hence no predation, which again goes against what we see in reality.

- So in Kreps/Wilson/Milgrom/Roberts, they introduce a small probability, ϵ, that an incumbent is a "jerk." That is, no matter what, the jerk will fight off an entrant just because he can. The other $1-\epsilon$ possible incumbents are rational. So, even with a finite number of periods in the game, in equilibrium, for any $\epsilon > 0$, however small, it can be shown that in the early stages of the game, both jerks and rational firms will fight off any entrants to try to signal that they are jerks. Eventually, beyond some period T, we will reach the equilibrium in Milgrom/Roberts with either entry/accommodation or no entry, but at least in the early stages, we have the possibility of equilibrium predation. Problem solved? Maybe.

24 Lecture 24: November 22, 2005

24.1 More on Predation and Entry Deterence

Limit Pricing or Limit Quantity Setting

- Consider a simple Stackelberg setup with firm 1, the incumbent, moving first and the entrant, firm 2, moving second. The entrant faces a fixed cost while the incumbent does not.
- We could get either accommodated entry as we have seen before if the fixed costs are small enough. However, as in G-24.1, the fixed costs for the entrant are so large that the only equilibrium is for firm 1 to set the monopoly quantity and for firm 2 to stay out. This is called "blockaded entry."
- A third case is when firm 1, the incumbent sets a quantity off his best response function that is just large enough to make the entrant indifferent from entering and staying out. This is called Limit Quantity setting. (Equivalently, setting a low price would achieve the same outcome and is called Limit Pricing). The incumbent still needs a way to bind his hands about his non-equilibrium response quantity which he may achieve using some sort of bundling strategy.

Milgrom and Robert

- In the paper by Milgrom and Roberts, they consider an entrant and an incumbent monopolist who is one of two types. Either the incumbent is a high type with high marginal costs, c_H, or he is a low type with low marginal costs, c_L. See G-24.2. We end up at either E_1 or E_2 depending on the incumbent's type.
- Suppose the entrant does NOT know the type of the incumbent. Assume:

$$\pi_H^E > F > \pi_L^E,$$

that is, the profits of the entrant if he faces a high type are greater than his fixed costs of entry which, in turn, are greater than the profits he would receive if he enters against a low type.

- Suppose demand is $P(Q) = a - Q$ and the entrant's prior probability that the incumbent is a low type is:

$$Pr(c = c_L) = \lambda.$$

- Thus profits of the monopolist incumbent can be written:

$$\pi^I(p, c) = (p - c)(a - p),$$

which has zeros at $p = c$ and $p = a$. See G-24.3. The profits of the low and high types are shown and the optimal price is also shown. The high type sets a higher (lower) price (quantity) than the low type.

- So what happens? If the entrant knew the incumbent's type, he would enter against high types and stay out against low types. The high type (high cost) monopolist may have an incentive to falsely signal that he is a low type and keep the entrant out. The incumbent can signal the entrant by setting his price. This is stage 1, a price choice by the incumbent. In stage 2, the entrant observes the incumbent's price signal and then decides whether or not to enter. The high type monopolist wants to signal he's a low type. This threatens the low type monopolist because, if the entrants know this stuff goes on, he may truthfully set his optimal price but still face entry because the entrant thinks he is a faking high type. In addition, if λ is small enough, ie the entrant thinks there is a good chance that he's facing a high type, he may enter no matter what the signal is!

- Suppose in stage one, a high cost monopolist signals with a price, P_H, and a low cost monopolist signals with a price, P_L. The entrant's strategy is a mapping:

  $$e : \Re \mapsto \{enter,\ stay\ out\}.$$

  We cannot impose SPNE because there is only one subgame (the whole game) since the entrant doesn't know if he's facing a low or high type given the incentives described above. Thus we seek a Perfect Bayesian Equilibrium (PBE). Thus, we also need a belief structure mapping observed prices (signals) into posterior beliefs:

  $$\mu(p) : \Re \mapsto [0, 1].$$

  Again, $\mu(p)$ is the posterior probability that the entrant thinks he's facing a low type. $\mu(p)$ must be consistent with the prior, the strategies, and Bayes rule. If we observe, $P = P_L$, we must set $\mu(p) = 1$. If $P = P_H$, then $\mu(p) = 0$. But if $P \neq P_L$ or P_H, then we are off the equilibrium path and anything is possible (see refinements literature).

- Consider some possibilities:

  - (1) If the high and low types always choose the same price, we have a pooling equilibrium and $\mu(p) = \lambda$.
  - (2) If the high and low types randomize between prices, we need to use Bayes rule to update after the signal.
  - (3) In a separating equilibrium where the low and high types signal with different prices, the authors show that the high type will always find it optimal to set its optimal (monopoly) price since it knows it will face entry so it might as well extract as much as it can while it lasts. A low type will have to signal with a slightly lower price than what is optimal to ensure than no one enters. Note under this case, we get precisely the SAME result as under the full information setting. The high types always face entry and the low types never do. However, since the high type's price is the same and the low type's price is strictly lower, this seems like a welfare improvement for the consumer.

25 Lecture 25: November 29, 2005

25.1 Antitrust Consequences of Ties and Foreclosure

- We now consider situations where a monopolist in one market tries to leverage that market power into another market via a tie or an exclusive deal to shut out, or foreclose, a competitor.
- Examples British Airways versus and Virgin Airlines and 3M versus Lepage. In the latter, 3M tried to use their monopoly in the branded tape market to foreclose Lepage out of the market for generic tape. 3M tried to make deals with, eg, Staples, that said we'll give you a deal on branded 3M tape if you also only buy your generic tape from us as well. Lepage filed suit and won.
- Is it possible to leverage monopoly power from one market to another? Are there efficiency reasons to tie? Are there exclusionary motives?
- There are three areas of research that has tried to give a theoretical answer to these questions.

1. Costless Exclusion

- In contrast to predatory pricing, which is costly for the incumbent, costless exclusion involves excluding an opponent without incurring a cost.
- Suppose we have a monopolist in market A who also produces product B where he competes with many rivals. The monopolist ONLY does linear pricing in market A. All firms in market B (including the monopolist) produce at constant marginal cost, c.
- See G-25.1. Pretie, the monopolist sets the monopoly price in market A and the competitive price in market B (along with all other firms). The monopolist offers a tie in which he charges a slightly higher price for B in exchange for a lower price in market A and a contract to only buy A and B from him. We could have a Pure Tie or a Mixed Tie. Due to Matthewson/Winter, we know that there exists prices that are both optimal and feasible for the monopolist that will completely exclude (foreclose) his competitors. In fact, it's WELFARE IMPROVING. The reason is that the social gains from lowering the monopoly price are large while the social costs of raising the price in market B are small.
- Under a mixed tie, the social surplus rises, as does the monopolist's profits, but the monopolist does even better under a pure tie. Thus the monopolist has the Costless Exclusion Property. The tie is very effective at forcing foreclosure of the monopolist's rivals.
- HOWEVER, the monopolist can do even better under nonlinear pricing (2 part tariff), so if you allow for this, the logic above breaks down. There is only one monopoly rent which the monopolist will gain and there is no room to leverage that into another

market. Thus there would NOT be a motive to tie the two products. The monopolist can do better with a 2 part tariff. Arguments against this say, well maybe nonlinear pricing is not allowed or we place some other restriction on the contracting space. However, tying is a nested device within a 2 part tariff so it is a hard story to tell that a firm can tie but cannot engage a 2 part tariff.

- So maybe costless exclusion is not the best model.

2. Divide and Conquer Strategy

- Suppose a monopolist can simply pay a consumer to buy from him. The problem with this is that the payment would have to be VERY large to make the consumer agree to it. It would be too costly for the firm.
- This is a typical Chicago School argument. No sensible firm would agree to this.
- However, if we assume that the market has a lot of buyers and there is a minimum efficient scale for the firm that isn't too large, the authors (which?) show that it is possible for the firm to contract with part of the market and not another part. So the monopolist signs an exclusive deal with only part of the market.
- This divide and conquer strategy is what kills the Chicago argument because you sign up just enough people to profitably foreclose rivals if certain conditions are satisfied.

3. Quantity Forcing Contracts

- Suppose firms 1 and 2 produce differentiated products. Firm 1 gives a quantity discount to consumers if they purchase a minimum amount of their product. Since the products are (partial) substitutes, this forecloses firm 2.
- This is clearly a nonlinear contract and it yields multiple equilibria, some of which involve quantity forcing by one firm and exit by the other. However, they are usually harmful to the firms and probably dominated by other strategies.

26 Lecture 26: December 1, 2005

26.1 Repeated Oligopoly

- Repeated oligopoly is a subset of dynamic oligopoly games where a 1 period stage game (eg, Cournot or Bertrand competition) is repeated a certain number of times.

- Can we achieve a monopoly/collusive outcome in these games if they are repeated? Do mechanisms exist that will remove the self-defeating competitive component that usually drives firms away from the monopoly solution?

- Clearly, if we have explicit collusion / exclusive dealing, we can achieve the first best outcome but this case is both uninteresting and illegal from an antitrust point of view.

- One way of achieving a monopoly like outcome through non-cooperative behavior is through a **Kinked Demand Curve**. Consider G-26.1. Suppose there is a focal price, p^f, such that if firms charge a price above p^f, they lose the entire market, and if they undercut p^f, they make relatively small gains (unlike the Bertrand case where uncutting induced a large increase in market share). One reason why firms might think this is the demand curve they face is through "Meet the Competition" policies. Suppose all firms have a policy that they will meet their competitor's price if it is lower than their own. Now undercutting the focal price just means that ALL firms effectively are lowering their prices so you don't get any gains in market share. The focal price may be the monopoly price, but it may also be something else. Either way, we get the possibility of achieving the first best (collusive) outcome through non-cooperative behavior.

The Folk Theorem Applied to Repeated Oligopoly

- Suppose we have a repeated game with n players where each player's strategy is a choice, $x_i \in \Re_+$. A stage game "play" in period t is a set of strategies:

$$x^t = (x_1, x_2, \ldots, x_n) \in \Re^n_+.$$

Payoffs are symmetric:

$$\pi_i(x_i, x_{-i}).$$

- The game is played T times (where T could be infinite). In period t, strategies can depend on all previous plays of the game up to period $t-1$ but not on other firm's period t strategies. Thus the strategy space is a mapping,

$$s : \Re^{n(t-1)}_+ \mapsto \Re_+,$$

where we denote the history of play up to period $t-1$ as $h_{t-1} \in \Re^{n(t-1)}_+$.

- The total payoff to firm i:

$$\Pi_i = \sum_{t=0}^{T} \delta^t \pi_i(x^t).$$

- Notation 1. Denote π^M to be the static monopoly profits in the stage game which is achieved by a play $(x_1^M, \dots, x_n^M)$.

- Notation 2. Denote π_i^O to be the static equilibrium profits (oligopoly outcome) of the stage game for firm i which is achieved by a play $(y_1^O, \dots, y_n^O)$. Assume this set of strategies is UNIQUE! Key assumption.

- Suppose T is finite. If Subgame Perfect NE is our equilibrium concept, clearly, with no last period, by backward reasoning, the monopoly outcome is never achieved and the unique SPNE is the T period repetition of the static (oligopoly) equilibrium.

- Suppose T is infinite. Still assuming the unique equilibrium of the stage game, let:

$$\pi_1^* = Max_z \, \{\pi_1(z, x_2^M, \dots, x_n^M)\}.$$

  So π_1^* is the highest profits from an optimal deviation of firm 1 if everyone else plays the monopoly strategy.

- Consider the following strategy of every firm in every period: If h_{t-1} is such that x^M is followed in all previous periods, select $x_i^t = x_i^M$. Otherwise, select $x_i^t = y_i^O$ forever. This is clearly a Grim Trigger strategy because if there is even one small screw up, all firms play the static Nash forever. Note the threat point (all firms playing the oligopoly equilibrium in all periods) is also subgame perfect so we have sequential rationality.

- If we suppose that firms split the market evenly if they play the monopoly outcome, cooperation yields:

$$\Pi_i^{coop} = \sum_{t=0}^{\infty} \delta^t \frac{\pi^M}{n} = \frac{\pi^M}{n} \frac{1}{1-\delta}.$$

- If a firm decided to deviate, profits for that firm are:

$$\Pi_i^{dev} = \pi^* + \sum_{t=1}^{\infty} \delta^t \pi^O = \pi^* + \pi^O \frac{\delta}{1-\delta}.$$

- Cooperation dominates if:

$$\begin{aligned}
\frac{\pi^M}{n}\frac{1}{1-\delta} &> \pi^* + \pi^O \frac{\delta}{1-\delta} \\
\frac{\pi^M}{n} &> \pi^*(1-\delta) + \pi^O \delta \\
\frac{\pi^M}{n} - \pi^* &> \delta(\pi^O - \pi^*) \\
\delta &> \frac{\pi^M/n - \pi^*}{\pi^O - \pi^*}
\end{aligned}$$

  Here, I assumed $\pi^* > \pi^O$. So we need δ to be high enough (firms value future profits a lot) for cooperation to be sustainable.

- **Remark** Note we don't need the firms to divide the monopoly profits equally if they cooperate. It could just be that the firm that gets the smallest share, ie, the one who has the greated incentive to deviate, must satisfy the above condition.

27 Lecture 27: December 6, 2005

27.1 More on Repeated Oligopoly

- Recall the grim trigger strategy to sustain implicit collusion required that:

$$\frac{\pi^M}{n}\frac{1}{1-\delta} > \pi_1^* + \delta\pi^O\frac{1}{1-\delta}.$$

  Which we can rewrite as:

$$\frac{\delta}{1-\delta}\left(\frac{\pi^M}{n} - \pi^O\right) \geq \pi_1^* - \frac{\pi^M}{n}.$$

- This equation is really the HEART of the Folk theorem. We now consider when this inequality will be easy to satisfy and when it will not.

- Consider the rate of time preference, δ. If $\delta = 0$, firms are completely impatient so the inequality never holds and we cannot sustain the collusive outcome. If $\delta \to 1$, the expression, $\delta/(1-\delta) \to \infty$, firms are VERY patient, and the inequality always holds. Thus, $\exists$ a δ^* such that for $\delta > \delta^*$, collusion is supportable by the Grim Trigger.

- Also important in satisfying the inequality is the length of time between renegotiations (eg, how often do firms get to set their price). If this time period is long, the gain from deviation might be large because the punishment phase is pushed out into the future. In the time period is short, collusion will be easier to sustain.

- What about π^O, the oligopoly profits. This is called the **Folk Theorem Paradox**. As $\pi^O \downarrow$, it is EASIER to support collusion. Thus, when the stage game is more competitive, we get collusion more often! The reason is that the threat of punishment is more severe if the stage game is competitive. It can be shown that:

$$\delta^*_{bertrand} < \delta^*_{cournot}.$$

- In a Bertrand setting, $\pi^O = 0$ and $\pi_1^* = \pi^M$. Thus, our condition becomes:

$$\begin{aligned}
\frac{\delta}{1-\delta}\left(\frac{\pi^M}{n} - \pi^O\right) &\geq \pi_1^* - \frac{\pi^M}{n} \\
\frac{\delta}{1-\delta}\frac{\pi^M}{n} &\geq \pi^M - \frac{\pi^M}{n} \\
\frac{\delta}{1-\delta}\frac{1}{n} &\geq 1 - \frac{1}{n} \\
\frac{\delta}{1-\delta} &\geq n-1 \\
&\ \text{if } n = 2 \\
\delta^* \geq \frac{1}{2}
\end{aligned}$$

So for two firms with a Bertrand stage game, the critical discount factor is only a half!

- The condition also varies with n. It can be shown, $\exists\, n^*$, such that, for $n > n^*$, collusion is not supportable and if $n < n^*$, you could support collusion. The idea is that if you have too many firms, then you would have to share the monopoly rents among a lot of firms in the collusive phase so the gains are small. Deviation will probably be profitable.

- **Remark** Since this is implicit collusion, it would be hard to legally punish firms for playing best response strategies! But in reality, usually the government would not have the data to bring a case against firms that they suspect of implicitly colluding.

- **Remark** But, it should be noted, that along with the trigger strategy equilibrium, there is also the SPNE strategy of playing the stage game equilibrium in every period. What to do about the multiplicity?

- **Remark** What about our assumption that T is infinite. Does that make sense? We offer three alternative that allow for a finite T:

  - (1) Tirole: Add a probability p, such that the game continues into the next period. We then have some condition on $p * \delta$ which must be high enough to sustain collusion. Not compelling because p must goes to zero eventually when the sun explodes.
  - (2) Kreps, Wilson, Milgrom and Roberts: can sustain collusion with a finite T if there is some small probability that one of the firms is a tit-for-tat player.
  - (3) Benoit and Krishna: suppose the stage game has more than 1 equilibrium. Then $\exists$ a T^* and δ^* such that for $\delta > \delta^*$ and for $T > T^*$, ANY outcome in the individually rational set can be achieved as a SPNE (including the monopoly outcome). This is a fairly negative result because it offers NO PREDICTABILITY!

- **Remark** Abreu considers what the optimal punishment phase should be under various forms of competition. For Bertrand, grim trigger is best. But for Cournot, a stick/carrot punishment is optimal where if a firm deviates, you flood the market hurting everyone (the stick) but then you revert to collusion and kick out the deviator (the carrot).

- **Remark** Bernheim and Winston consider multimarket effects on the Folk condition. See literature.

- **Remark** With the Folk structure, we should NEVER see price wars since collusion is always optimal. But in reality, we do, so what explains this? We need to introduce some randomness to demand. Suppose there are booms and busts in the market. During booms, it turns out that deviation is MORE likely since the deviant has more to gain today and the discounted flow of tomorrow profits are low compared to today's booming market. Could even get countercyclical prices with this setup. This takes us into Green/Porter.

Green/Porter Model of Price Wars

- Suppose demand is random:

$$Q = 1 \; for \; p \leq 1, \; Q = 0 \; for \; p > 1 \text{ with probability } 1 - \alpha.$$

$$Q = 0 \; \forall \; p; \text{with probability } \alpha.$$

- Two firms producing a homogenous good simultaneously set their price in every period. Firms ONLY see their own prices and own demand.
- Suppose firm 1 set a price of 3/4 and the resulting demand is zero. This could be either because the market is in a bust period, or because firm 2 undercut firm 1. There is no way for firm 1 to know which state of the world he is in.
- Is there an equilibrium with 2 phases: (C)ollusive and (P)unishment, such that:
  - in the C phase, $p_1 = p_2 = 1$,
  - in the P phase, $p_1 = p_2 = 0$, and firms stay in this phase for T periods before reverting to the C phase.
- Does $\exists$ a T such that it is unprofitable to deviate?
- Let V^+ be the discounted expected value of a firm in the C phase and let V^- be the discounted expected value of a firm at the START of a P phase. It must be that:

$$V^- = \delta^T V^+,$$

ie the value of a firm in the punishment phase is zero for T periods and then V^+ disounted by δ, T times. Also,

$$V^+ = \underbrace{(1-\alpha)}_{boom}\Big(\underbrace{\frac{1}{2}}_{\pi \; today} + \underbrace{\delta V^+}_{future \; collusion}\Big) + \underbrace{\alpha}_{bust} \underbrace{(0 + \delta V^-)}_{today \; plus \; punish}.$$

- In equilibrium, we must have:

$$V^+ > \underbrace{(1-\alpha)(1 + \delta V^-) + \alpha(0 + \delta V^-)}_{gains \; from \; deviating}.$$

Which reduces to:

$$\delta(V^+ - V^-) > \frac{1}{2}.$$

Substituting in the condition above, we can solve for:

$$2(1-\alpha)\delta + (2\alpha - 1)\delta^{T+1} \geq 1.$$

- If $\alpha > \frac{1}{2}$, ie, the probability of busts are large, then $T = 0$ and $\delta \geq 1$, which is not possible. Thus, since the gains from supporting a collusive outcome are small, we can't satisfy the inequality.
- If $\alpha < \frac{1}{2}$, then for some T and δ close to 1, we can satisfy the inequality. Note, we want T to be as small as possible.
- So the Key Point is that in equilibrium, we get both C and P phases! Thus price wars can be consisent with this equilibrium.

28 Lecture 28: December 8, 2005

28.1 Size Distribution of Firms

Schmalensee

- Consider the following paradigm for the structure of markets:

  Market Conditions ⇒ Market Structure ⇒ Conduct/Behavior ⇒ Market Performance.

  Market conditions include demand, product type, public policy, supply conditions, and technology. Market structure includes the number of buyers and sellers, entry barriers, and vertical-integration. Conduct includes how pricing, advertising, and R&D decisions are made. And performance includes profitability, efficiency, innovation, and possibly employment.

- The old IO models all drew the arrows in one direction as above and considered INTERindustry studies. They considered regressions of, say, performance on market structure to determine if there was a causation between industry concentration and profitability.

- Schmalensee is skeptible of these types of regressions because the arrows could (and probably do) go both ways. This introduces feedback and endogeneity in our regressions. So all we can really say is that profitability and concentration may be correlations but we can't say anything about causation.

- The next direction for the literature was INTRAindustry studies of a bunch of firms across time (panel data studies).

- One finding of this literature is that the size distribution of firms turns out to be the SAME across different industries! See G-28.1. It appears to be log-normal. If you take the log of, say, firm sales, and plot it, you'll get a normal looking distribution (G-28.2).

- This motivates two questions:

  - (1) Why is the distribution the same across markets?
  - (2) Why is the distribution log-normal?

Gibrat

- **Gibrat's Law of Proportional Effect** Assume firm size in period 0 is X_0, a random variable. Firm growth rate in period t is:

$$g_t \sim iid.$$

Thus a small firm and a large firm have equal chances of growing by 10 percent. Strong assumption. However, this implies:

$$X_T = X_0(1 + g_1)(1 + g_2) \cdots (1 + g_T).$$

So,

$$log(X_T) = log(X_0) + \sum_{t=1}^{T} log(1 + g_t).$$

And since we have the sum of a bunch of iid random variables, $log(X_T) \rightarrow$ normal, by a CLT.

- The problem with Gibrat's law is that it says TOO much! The assumption that g_t is iid is too strong and can easily be rejected. Back to Schmalensee.

More Schmalensee

- Schmalensee looks at intraindustry level data and finds the following stylized facts:
  - (1) The distribution of firms is skewed (positively).
  - (2) Large firms tend to have low average growth rates and the variance of their growth rates tends to be lower compared with small firms.
  - (3) Old firms tend to have low average growth rates and the variance of their growth rates tends to be lower compared with young firms.
  - (4) Old and large firms have higher survival rates.
  - (5) Growth rates tend to be serially correlated.
- So what drives the similar size distribution of firms is the it's not this iid assumption of g_t ? Lucas provides one solution.

Lucas

- Bob says that the thing that all firms have in common that might be driving the similar size distribution across industries is access to managerial talent.
- Suppose there are two inputs to production: labor and capital and we have an index production function $g(n, K)$ which is CRTS and concave. True output, however, is as follows:

$$f = x * g(n, K),$$

  where x is the skill level of the manager of the firm.
- See G-28.3 for the true production function as it varies over laborers, holding capital fixed. This induces a strong correlation between the skill level of the manager and the size of the firm.

- So in G-28.4, we consider the spectrum of skill levels available to a firm. Those above x^* become managers, and those below, become workers. This common distribution is what is driving the similarity of the size distribution across industries.
- **Remark** As a country grows, or technology improves, x^* should shift to the right which increases the mean skill level of the managers. Thus the size of the firms should also grow.
- We next consider another model of an industry to explain the size distribution of firms based on signal extraction issues.

28.2 Jovanovic

- Suppose there is a certain size such that if a firm receives a negative shock and ends up below this size, they exit. See G-28.5. The small firm that has approached the cutoff and then receives an adverse shock will EXIT the industry, so our data set on growth rates will truncated. This will bias UP the growth rate of firms because g_t is now conditional on survival. This might lead to a false rejection of Gilbrat. But this wasn't Jovanovic's main point.
- Suppose firms have an unknown "suitability" for operating in a given industry, and they receive noisy signals of this characteristic.
- Suppose there is a large mass of firms who start out symmetrically. They have access to the same marginal cost structure, $c(q)$, with $(c' > 0, c'' > 0, c(0) = 0)$, where in any given period, the TRUE cost of a firm is:

$$x_{it} * c(q),$$

where x_{it} is a random variable satisfying:

$$x_{it} = \xi(\eta_{it}),\ \xi' > 0.$$

In turn,

$$\eta_{it} = \theta_i + \epsilon_{it},\ \theta_i \sim N(\overline{\theta}, \sigma_\theta^2),\ \epsilon_{it} \sim N(0, \sigma_\epsilon^2),$$

where θ_i is nature's draw of a firm's type, unobserved by a firm. η_{it} is potentially observed.

- Any firm that enters bears the one time cost, $K > 0$.
- Production involves forgoing an opportunity cost of $W > 0$.
- There is a known demand structure, $P_t(Q_t)$.
- An operating firm faces a price, P, and constructs an expected value, x^e, before choosing his quantity, q.

- In the first period, a firm's best guess of η_{it} is $\overline{\theta}$ and thus $x^e_{it} = \xi(\overline{\theta})$. Firm's then choose q. Later they observe η_{it} but don't know if a good draw, say, is coming from a high θ or a high ϵ. Signal extraction.

- They then update η based on the variances of θ and ϵ as in Lucas Islands.

- As time goes on, firms get more and more information about their true type, θ_i. Eventually, firms will exit if they realize they really aren't "suited" for producing in this industry.

- More next time.

29 Lecture 29: December 13, 2005

29.1 Size Distribution of Firms - Jovanovic

- Recall our model from last time. Costs:

$$x_{it} * c(q),$$

where x_{it} is a random variable satisfying:

$$x_{it} = \xi(\eta_{it}),\ \xi' > 0.$$

$$\eta_{it} = \theta_i + \epsilon_{it},\ \theta_i \sim N(\overline{\theta}, \sigma_\theta^2),\ \epsilon_{it} \sim N(0, \sigma_\epsilon^2),$$

where θ_i is nature's draw of a firm's type, unobserved by a firm.

- Any firm that enters bears the one time cost, $K > 0$.
- Production involves forgoing an opportunity cost of $W > 0$.
- There is a known demand structure, $P_t(Q_t)$. No uncertainty here. Only in costs.
- Firms are completely competitive price taking firms that may differ in size. The concentration indices will be meaningless as a result.
- In period t, firms first estimate x_{it}^*, its productivity parameter and then maximize:

$$\pi(q) = p_t q_t - c(q) x_{it}^*.$$

They produce q_t and then afterwards, observe their true x_{it}. This is a signal regarding their true type, θ_i, but since there is also an ϵ_{it} they cannot completely untangle the two components.

- Firms use their estimate of θ_i to form a new estimate, $x_{i,t+1}^*$ for next period.
- If a firm has been operating for more periods, they have more information regarding their true type so have less uncertainty.
- Normality implies that a sufficient statistic for x_{it}^* is,

$$\overline{\eta}_{it} = \frac{1}{n} \sum_{\tau=1}^{n} \eta_{i\tau},$$

and n, the age of the firm. So you don't need the entire distribution of your experiences, just the average experience, as well as your age. This simplifies (a bit) the computation.

- When firms generate their estimates of x_{it}^*, they will use some weighted average of $\overline{\theta}$ and $\overline{\eta}_{it}$ where the weights will depend on the variances.

- Firms, in addition to choosing q, must decide to enter if they are not in the industry (and bear a cost, K) or if they are already in, they can decide to exit (and gain a benefit, W). A firm in period t with productivity x, age n, and facing price p will get:

$$V(x,n,t;p) = Max\ \{pq - c(q)x\} + \delta \int_z Max\{W, V(z,n+1,t+1,p)\}dz.$$

- A firm exits if $V(x,n,t;p) \leq W$. Thus for some threshold level of x, firms will decide to exit. Ie, if x is too high, costs are too high so a firm will want to exit.

- **Remark** This is a clear violation of Gibrat's law: growth rate are NOT independent across time. A sequence of bad draws for a firm makes it more likely that the firm will get more bad draws (ie, their type really is shitty) and they will exit.

- **Remark** Firms are less and less likely to respond to shocks to their productivity because since their information is very good as they get older regarding their true type, any shock must be coming from ϵ. Thus the variance of firm growth rates will shrink over time as firms "learn" about their type.

- **Remark** Small firms (high θ) will eventually exit. Older firms tend to be large (low θ) since they have survived over time and end up producing more and more.

Hopenhayn Paper

- In Jovanovic, there is NO stationary solution since there is always learning going on. Instead Hopenhayn (Hop) says that the productivity parameter is still a random variable, but it's markov. I can tell you the entire distribution of tomorrow's productivity parameter if you just give me yesterday's true value.

- In Jovanovic, over time, you get less and less entry because the surviving firms become larger and the market becomes more concentrated. Empirically this doesn't hold up so Hop's paper tries to solve this puzzle.

- Hop says, suppose profits are:

$$\pi(\phi, p, w, n),$$

  where $\phi_t \sim F(\cdot|\phi_{t-1})$, our random productivity parameter (this time bigger is better), p is price, w is wage, and n is the number of workers.

- Assume for $\phi' > \phi''$,

$$F(\phi|\phi') < F(\phi|\phi'').$$

- The key result is that $\forall\ \epsilon > 0,\ \exists\ t \ni F^t(\epsilon|\phi) > 0$ where F^t is the distribution of ϕ_t given ϵ. In english, this means that no matter how well I'm doing today, there is always going to be some positive probability that, in the future, my productivity is going to be very low, and I will be forced to exit. Thus, all firm are finitely lived! Eventually everyone gets bad news and exits.

- Thus in Hop, we get entry and exit in the long run in equilibrium, which is more inline with empirical observations.

Dunn, Roberts, and Samuelson

- They consider testable implications of Jovanovic. The distribution of growth rates of firms is truncated from below by those that decide to exit the industry. Using very good census data, they estimate the growth rates of surviving firms, the failure rates of firms, and the unconditional growth rates. See paper.
- Read Jovanovic before the Fields ... Vincent may ask on this.