Microeconomics
Exam Review

May 31, 2002

1 Michaelmas Mock Revision Notes

1.1 Properties of major functions

- Unconditional Input Demands, $x(p, w)$.
  - Decreasing in the cost of inputs, w.
  - Homogeneous of degree 0 in (p, w).
- Supply Function, $y(p, w)$.
  - Increasing in output price, p.
  - Homogeneous of degree 0 in (p, w).
- Profit function, $\pi(p, w)$.
  - Non decreasing in prices, p. Non increasing in input prices, w.
  - Homogeneous of degree 1 in prices, p.
  - Convex in prices, p.
  - Continuous.
  - Look for a Positive Semidefinite Hessian Matrix. (All leading P.M. Dets are non-negative)
- Cost Function, $C(w, y)$.
  - Non decreasing in input prices, w.
  - Homogeneous of degree 1 in input prices, w.
  - Concave in input prices, w.
  - Continuous.
  - Look for a Negative Semidefinite Hessian Matrix. (1st: Negative, 2nd: Positive, 3rd: Negative, ...)
- Indirect Utility Function, $V(p, m)$.
  - Non increasing in prices, p. Non decreasing in income, m.
  - Homogeneous of degree 0 in (p, m).
  - Quasi-Convex in prices, p.
  - Continuous.
- Expenditure Function, $e(p, u)$.
  - Non decreasing in prices, p.
  - Homogeneous of degree 1 in prices, p.
  - Concave in prices, p.
  - Continuous.

1.2 Duality

- Hotelling Lemma:

$$\frac{\partial \pi(p,w)}{\partial p} = y(p,w).$$

$$\frac{\partial \pi(p,w)}{\partial w_i} = x_i(p,w_i).$$

- Shephard's Lemma:

$$\frac{\partial C(w,y)}{\partial w_i} = x_i(w_i,y).$$

$$\frac{\partial e(p,u)}{\partial p_i} = h_i(p,u).$$

- Roy's Identity:

$$x_i(p,m) = -\frac{\partial v(p,m)/\partial p_i}{\partial v(p,m)/\partial m}.$$

- Slutsky:

$$\frac{\partial x_1}{\partial p_1} = \frac{\partial h_1}{\partial p_1} - \frac{\partial x_1}{\partial m} x_1.$$

2 Michaelmas Mock Revision Notes

2.1 Hotelling, Price Competition, and Entry

- Trade off between price competition and entry. The tougher the price competition, the less entry that will occur because each successive entrant drives down the profits per firm. However, less entry in the industry implies that profits will be higher and should attract new entrants. In equilibrium, there is a subtle balance between price competition and entry.

- Hotelling's 1929 Error. Technically, the problem with Hotelling's original model was the assumption of linear costs which could result in discontinuous jumps in the demand schedule as shown in the notes. Invoking quadratic cost curves (umbrellas) eliminates this possibility. However, the fundamental error is not just in the discontinuities. Having the linear costs meant that in equilibrium, two firms would choose to locate right next to each other on the line and split the market evenly. He referred to this as the "Principal of Minimal Differentiation." Invoking quadratic costs yielded the exact opposite conclusion: Two firms choose to locate at exactly the opposite ends of the line and thus the "Principal of Maximal Differentiation." The two forces at work here is first the position of the marginal man. Two firms located along the line at distinct points would both have an incentive to move towards their neighbor, thus shifting over the marginal man and gaining sales volume. However (the second force), moving closer to your neighbor also makes the products more substitutable (less differentiated) and thus drives down prices and profits. In an extreme case, with bertrand price competition, it is easy to see that two firms would want to be as far away from each other as possible because if they had to compete, prices would be driven down to marginal cost and both firms would make zero profit. Hotelling ignored the price competition idea in his original analysis and thus didn't see how a simple adjustment to the cost schedule could yield precisely the opposite results. The reason that the change to the cost schedule yields this idea is because moving away from your neighbor now results in higher profits because of the product differentiation effect. Before (with linear costs), this effect was too small and thus Hotelling's original conclusion. With quadratic costs, it is now beneficial to the firms to move away from each other, losing the sales volume from the shifting marginal man, but gaining positive profits from the increased product differentiation.

2.2 Bundling Properties

- Complete Extraction: Yes, if all consumers have zero consumer surplus.

- Exclusion: Yes, if no consumers are consuming when the production cost of the good exceeds the consumer's reserve price for that good.

- Inclusion: Yes, if no consumers are NOT consumeing who have a reserve price which exceeds the production cost of the good. Also referred to as Allocationally Efficient.

- Distributionally Efficient: Yes, if there does not exist any gains from trade between the consumers.

CV and EV

- Consider $P^1 < P^0$.

	P^0	P^1
u^0	m	$m - CV$
u^1	$m + EV$	m

2.3 Hessian Properties

	Properties	"	Constrained Optimization	"
	Convex	Concave	Minimization	Maximization
Class	Pos. S-Def.	Neg. S-Def.	Pos. S-Def. Board. Hess.	Neg. S-Def. Board. Hess.
H_1, H_2, H_3	+ + +	- + -	- - -	+ - +

2.4 GeoSeries

- One:

$$\sum_{i=0}^{\infty} x^i = \frac{1}{1-x}.$$

- Two:

$$\sum_{i=1}^{\infty} i x^{i-1} = \frac{1}{(1-x)^2}.$$

3 Final Exam Revision Notes

3.1 Quasi's

- A function f is concave if:

$$f(\alpha x + (1-\alpha)y) > \alpha f(x) + (1-\alpha)f(y).$$

- A function f is Quasi-concave if:

$$f(\alpha x + (1-\alpha)y) \geq min\{f(x), f(y)\}.$$

- A function f is convex if:

$$f(\alpha x + (1-\alpha)y) < \alpha f(x) + (1-\alpha)f(y).$$

- A function f is Quasi-convex if:

$$f(\alpha x + (1-\alpha)y) \leq max\{f(x), f(y)\}.$$

3.2 Taxes and Subsidy Equations

- Consider two industries: M (Manufacturers) and F (Food) with prices P_M and P_F. Industry M is labor intensive while industry F is capital intensive. Labor is paid wage, w. Capital is paid at rate, r. Starred variables represent the change over time. Consider subsidies (negative taxes), S_M^* and S_F^* on the M and F industries. Thus we have two results (via Stolper - Samuelson):

- Effect on Commodity Prices:

$$P_M^* - P_F^* = -\frac{\sigma_S}{\sigma_D + \sigma_S}(S_M^* - S_F^*).$$

- Effect on Factor Prices:

$$w^* - r^* = \frac{1}{|\theta|}\frac{\sigma_D}{\sigma_D + \sigma_S}(S_M^* - S_F^*).$$

- For example, a subsidy on the M industry raises the return on labor, w (the factor used intensively in that industry) relative to the return on capital, r. This also has the effect of lowering the commodity price in the M industry relative to the F industry. The magnitude of these two effects will all depend on the elasticities of substitution between the commodities on the supply and demand side.

- Limiting cases. If $\sigma_D \longrightarrow 0$ implies that M and F are perfect complements. The demand curve is very steep which means that a change in the price ratio will not have a large impact on the consumption choice between M and F. In fact, in the limit, the effect is zero and the subsidy is completely passed on to the consumer and the relative efficiency of labor and capital is unchanged.

- If $\sigma_D \longrightarrow \infty$ implies that M and F are perfect substitutes. The demand curve is very shallow which means that a change in the price ratio will have a large impact on the consumption choice between M and F. In fact, in the limit, the subsidy has no effect on consumer prices but rather the entire effect is captured in the higher return to labor.
- In sum, the greater is the price cut passed on to consumers, the less is the gain to labour.

3.3 Notes on Key Articles

3.3.1 Schmalensee - 3rd Degree Price Discrimination

- Robinson shows geometrically that if a single price monopoly selling in two markets under constant costs is allowed to discriminate between them, total output is unchanged if both markets have linear demand curves.
- All the formal analysis so far rests on the assumption that $q_i(p^*)$ and $q_i(p_i^*)$ are positive for all i. This assumption is clearly rather strong however: some weak markets may not be served at all by a single price monopoly even though a discriminating monopolist could profitably make sales to them.
- In the linear demand case, we have a drop in Marshallian welfare. In general, unless output increases, movement from single price to discriminating monopoly causes a fall in W, thus a net efficiency loss.
- Total welfare effect:

$$\frac{dW}{dt} = [p^* - c][\frac{dQ}{dt} + \sum_i^N [p_i - p^*]q_i'p_i'.$$

  The first term is the welfare gain from the output change in a distorted market. The second term reflects the efficiency cost of distributing total output inefficiently amoung buyers by driving marginal valuations apart.
- The net change in Welfare can thus only be positive if total output expands or only if the increase in sales to the weak market exceeds the drop in sales to the strong market.
- Important note: All this is qualified to some extent by the possibility that such discrimination makes it profitable to sell to markets that would not be served at all under single price monopoly. If discrimination makes possible a large volume of such new sales, it can lead to an increase in welfare even if total sales to previously served markets fail to expand.

3.3.2 Rubinstein - Bargaining

- Equilibrium in an infinitely repeated game:

$$(x, y) = \left(\frac{1-\delta_2}{1-\delta_1\delta_2}, \frac{\delta_2(1-\delta_1)}{1-\delta_1\delta_2}\right).$$

- The above result has the intuitively appealing feature that the more impatient a player, the smaller his share of the pie. It also has the less attractive feature that there is an advantage to "moving first." For example if $\delta_1 = \delta_2 = \delta$, the payoffs reduce to:

$$(x, y) = \left(\frac{1-\delta}{1-\delta^2}, \frac{\delta-\delta^2}{1-\delta^2} \right) = \left(\frac{1-\delta}{(1-\delta)(1+\delta)}, \frac{\delta(1-\delta)}{(1-\delta)(1+\delta)} \right) = \left(\frac{1}{1+\delta}, \frac{\delta}{1+\delta} \right).$$

  So for any given discount level, player I has a first mover advantage and realizes a higher payoff.

- The first mover advantage can be removed by removing the identity of the proposer in each period by tossing a coin (Binmore 1980). This advantage can also be removed by introducing a time delay between periods, Δ. Write the discount factor as δ^{Δ}. Then the limit as $\Delta \to 0$, the equilibrium share of player I is:

$$x = \lim_{\delta \to 0} \frac{1-\delta^{\Delta}}{1-\delta_1^{\Delta}\delta_2^{\Delta}} = \frac{ln\delta_2}{ln\delta_1 + ln\delta_2}.$$

  If $\delta_1 = \delta_2$, then $x = \frac{1}{2}$.

3.3.3 Hendricks and Porter - Oil Tract Auctions

- An easy extension of the result, which is of importance empirically, is to let the support of v include negative values so that for some $x > 0$, we have $-x \leq v \leq 1$. This allows for the fact that some tracts are unprofitable. The informed player will not, at equilibrium, bid on these tracts. The uninformed player, who still makes zero profit overall, makes strictly negative profit on bids for these tracts. We take advantage of presence of tracts of this kind when splitting the sample of outcomes in empirical testing.

3.3.4 D'Aspremont, Gabszewicz, and Thisse - On Hotelling

- One should expect intuitively that product differentiation must be an important component of oligopolistic competition. It seems to be clear that oligopolists should gain an advantage by dividing the market into submarkets in each of which some degree of monopoly would reappear.

3.3.5 Adams and Yellen - Bundling

- Commodity bundling can be profitable even when the complementarity arguments about consumption in bundles fails. We show that the profitability of commodity bundling can stem from its ability to sort customers into groups with different reservation price characteristics and hence to extract consumer surplus.

- In some circumstances, bundling is just as profitable as Piguvian price discrimination of the first degree. In most circumstances, it is more profitable than simple monopoly pricing.

- In words, the bundle is consumed by those who not only derive positive consumer surplus from the purchase of the bundle, but also derive more surplus from the bundle ($R_B - P_B$) than they would from the purchase of either component separately.
- In general, whenever the exclusion requirement is violated in a pure bundling equilibrium, mixed bundling is necessarily preferred to pure bundling.
- Bundling example - a la carte menu versus set menu.
- Bundling often extracts more consumer surplus than simply monopoly pricing and does so with far less information.
- In sum, commodity bundling generally leads to welfare losses when compared with perfect competition. But this does not imply that banning package selling per se decreases the burden of monopoly.

3.3.6 Spence - Job Market Signalling

- If one believes I will be in the essay market repeatedly, then both the reader and I will contemplate the possibility that I might invest in my future ability to communicate by accurately reporting the content of this essay now. On the other hand, if I am to be in the market only once, or relatively infrequently, then the above-mentioned possibility deserves a low probability.
- Applications: job markets, admissions procedures, promotion in organizations, loans and consumer credit.
- This essay is about the endogenous market processs whereby the employer requires information about the potential employee which ultimately determines the implicit lottery involved in hiring, the offered wages, and in the end, the allocation of jobs to people and people to jobs in the market.
- Individuals are assumed to select signals so as to maximize the difference between offered wages and signaling costs.
- A Critical Assumption: Costs of signaling are negatively correlated with productive capability.
- Sex can be used as a signal if and only if at some point in time the education choices available to men and women were different. Men and women could then settle into different signaling equilibria and thus sex could then be used as a signal to employers.

3.3.7 Akerlof - Lemons

- There may be potential buyers of good quality products and there may be potential sellers of such products in the appropriate price range; however, the presence of people who wish to pawn bad wares as good wares tends to drive out the legitimate business. The cost of dishonesty, therefore, lies not only in the amount by which the purchaser is cheated; the cost also must include the loss incurred from driving legitimate business out of existence.

- Greshams Law. Bad money drives out good money. When two media of exchange come into circulation together, the more valuable will tend to disappear. Where by legal enactment a government assigns the same nominal value to two or more forms of circulatory medium whose intrinsic values differ, payment will always be made in that medium of which the cost of production is least, and the more valuable medium will tend to disappear from circulation. In the case where the combined amount in circulation is not sufficient to satisfy the demand for currency, the more valuable medium will simply run to a premium.

3.3.8 Krugman - Causes of Trade

- This paper develops a simple model in which trade is caused by economies of scale instead of differences in factor endowments or technology.
- Pricing rule:
$$p_i = \lambda^{-1} v'(x_i/L).$$
If the number of goods produced is large, each firm's pricing policy will have a negligible effect on the marginal utility of income, so we can take λ as fixed.
- Chamberlin's Tangency Solution: $MR = MC$ and $AR = AC$.
- The PP (pricing policy) curve slopes upwards is an assumption which comes from the fact that the elasticity of demand rises when the price of a good is inccreased. This seems plausible.