Industrial Organization
Michaelmas Term

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1 Week 1

- Three Divisions of Microeconomics: Labor Markets, Capital Markets, and Product Markets (IO).
- Bain Paradigm: Structure $\longrightarrow$ Conduct $\longrightarrow$ Performance.
- Led to running regressions of structure on performance and concluded a positive relationship but further study produced conflicting results. Structure measured by industry concentration. Performance measured by profits as a fraction of assets. Conduct is relatively hard to measure.
- If there is a positive correlation and firms are making profits, there is an incentive for entry. Barriers though might impede entrants. Game Theory was the next logical step in solving this complicated puzzle.

1.1 Game Theory

- Game Theory: 3 objects. 1) Players 2) Strategies for each player 3) Payoff function which maps the set of strategies of all agents into agents i's payoff.
- Nash Equilibrium (NE): A set of strategies, one for each player, such that, given the strategies of its rivals, each player is using a strategy that maximizes his payoff (Optimal Reply or OR)
- In an example of firm price competition: The price the firm charges is NOT the strategy. The price the firm sets is the firm's action. A strategy, in this case, would be more of a "pricing plan" or a system of relative prices.
- The essence of non-cooperative game theory is that there are no binding agreements.
- Some games have no pure strategy NE. Hence the necessity for mixed strategies where players have probabilities of playing individual strategies.
- There may also be several optimal replies for a player in pure or mixed strategies.

2 Week 2

2.1 More Game Theory

- Usually there is are embarrassingly many NE, thus the need to define a new condition, Perfectness.
- NE are a subset of all possible equilibria in a game. Perfect equilibria are a further subset of NE.
- Subgame Perfection: A subgame perfect nash equilibrium (SPNE) is a set of strategies that form a NE which induce a NE in every subgame.
- Subgame: if we can find a node, such that, the game tree starting from this node leads us to information sets which incorporate nodes of this tree only, then the game hanging from this node is a subgame.
- Selten's Example: Refer to game tree in notes. **[G-2.1]** The idea is that there are no subgames in the game so all NE are SPNE. The NE that is chosen is not "sensible." One of the player's actions is independent of his payoff. If a certain node is reached, the player should be guided by his expected payoff as a function of all previous plays.
- The only way out of this situation is 1) Trembling hand perfect equilibrium or 2) Sequential Equilibrium which involves equiping players with both strategies and beliefs.

3 Week 3

3.1 The Short Run - Price Competition

- One Shot Games. In the Cournot type game, $P > MC$ but falls as more firms enter. In Bertrand type games, $P = MC$ for 2 or more firms in the market.
- Dynamic Games.
- Repeated Games or "Super Games." Structure: Take a one shot game and call it G. G is the "constituent game" or the "stage game". Players play G at periods $t = 1...T$ or $t = 1...\infty$.
- Payoff Function:

$$\sum_{t=1}^{T} \delta^t \pi_t.$$

  Where T could be ∞ and $\delta \in (0, 1)$.

- A strategy in G_T (or G_∞) is a function that prescribes an action in G at time t as a function of all actions taken a periods $1, 2, ..., t-1$. Thus there is a very large strategy space based on all the possibilities the game could have taken.
- Folk Theorem: Consider the bertrand game as the constituent game for the sake of this illustration. Let each player use the strategy: Set $p_t = p^m$ (the monopoly price) in each period unless a rival has set $p < p^m$ in some earlier period in which set $p_t = MC$ (marginal cost) in all future periods. Thus Folk states that for δ sufficiently high, the cooperative solution can be supported as a NE of G_∞.
- In a duopoly, $\delta > \dfrac{1}{2}$ is sufficient. In the n firm case, $\delta > 1 - \frac{1}{n}$ is sufficient.
- To show this consider the profit stream with and without cooporation.
- With Cooperation:

$$\pi = \frac{1}{n}\pi^m + \delta\frac{1}{n}\pi^m + \delta^2\frac{1}{n}\pi^m + \delta^3\frac{1}{n}\pi^m + ... = \frac{1}{n}\pi^m \sum_{i=0}^{\infty} \delta^i = \frac{1}{n}\pi^m \frac{1}{1-\delta}.$$

- Without Loss of Generality (WLOG), let the first deviation in the noncooperative case take place at $t = 1$. The deviation profit stream is:

$$\pi' \leq \pi^m + 0 + 0 + 0 + ... = \pi^m.$$

  This is because in the deviation period, the firm takes the whole market and gets almost monopoly profit but afterwards losed all profits because $p = MC$.

- So to sustain cooperation, (ie to make deviation unprofitable):

$$\begin{aligned}\pi^m &< \frac{1}{n}\pi^m\frac{1}{1-\delta}.\\ 1-\delta &< \frac{1}{n}.\\ \delta &> 1-\frac{1}{n}.\end{aligned}$$

- A trigger strategy specifies two actions plus a rule for switching between the two.
- The strategy used above is a Grim trigger strategy because one deviation leads to never going back to cooperation (no second chances).
- So the main idea here is that not only can we support p^m, we can support any price, $MC \leq p \leq p^m$ as a NE.

3.2 Maskin - Tirole Model

- Consider a duopoly with a homogeneous product, demand schedule: $D(p_1, p_2) = 1 - min(p_1, p_2)$. $MC_1 = MC_2 = 0$. Assume unlimited capacity.
- Strategy space: firm 1 chooses its price in odd numbered periods and holds it for 2 periods.
- Strategy space: firm 2 chooses its price in even numbered periods and holds it for 2 periods.
- Assume that the price is chosen from a discrete net of prices, $p_i = \frac{i}{n}$. So here, just let $p_i = \frac{i}{6}$.
- We will consider equilibria in Markov strategies, or firm 1's strategy depends only on firm 2's stategy in the last period. So p_t is a function of p_{t-1} only.
- Payoff Function = Net present value (NPV) of profits flows: $\pi = \sum_{t=1}^{\infty} \delta^t \pi_t$.
- Based on the demand function, prices can be anywhere from 0 to 1. Since $p_i = \frac{i}{6}$, i ranges from 0...6. If $i = 6, 5, 4,$ or 3, the Markov strategy (Optimal reply) is always to set $i = 3$ because this yields the monopoly price, $p^m = \frac{1}{2}$. If $i = 2$, the other firm undercuts and sets $i = 1$. If $i = 1$, the other firm has a mixed stratgy to either match the price and set $i = 1$ or to jump back up the monopoly price and set $i = 3$.
- Thus, along the equilibrium path of the game, $i_1 = i_2 = 3$ for all periods. Suppose 1 firm deviates by setting $p = 2$ in some period. We now examine the equilibrium path in the subgame which follows this deviation. (See notes) [**G-3.1**].

4 Week 4

- Cartel Stability. Two models: 1) d'Aspremont et al. Price Leadership Model. 2) Green/Porter Secret Price Cutting model.
- The following two models provide ways in which to uphold a cartel and eliminate the necessity for one firm to undercut the price and capture a larger portion of the market.

4.1 Price Leadership Model (d'Aspremont et al.)

- Assume perfect information, n firms selling a homogeneous product.
- Let α represent the fraction of firms that are cartel members. Therefore $(1-\alpha)n$ is the number of firms that are "fringe" firms.
- Industry demand schedule: $Q = D = n(1-p)$.
- All firms have the same cost schedule: $C = \frac{1}{2}q^2$ and thus $MC = q$.
- The way this would work, is the cartel members would choose the price level, p. The fringe firms would follow suit and determine their quanitity produced by setting their $MC = p$. Thus, $q_f = p$. The cartel then faces the residual demand schedule after the fringe firms produce their quantities.
- Since there are $n(1-\alpha)$ fringe firms each producing $q_f = p$, total fringe production is,

$$Q_f = (1-\alpha)np.$$

Thus each of the fringe firms have profits:

$$\pi_f = q_f p - C = p^2 - \frac{1}{2}p^2 = \frac{1}{2}p^2.$$

- Hence total cartel output,

$$Q_c \equiv \text{Market demand minus fringe production} = Q(p) - Q_f(p).$$

$$Q_c = n(1-p) - n(1-\alpha)p.$$
$$Q_c = n[1-(2-\alpha)p].$$

- Total cost for an individual cartel member if output is allocated equally among members:

$$C_{i_c} = \frac{1}{2}\left(\frac{Q_c}{n\alpha}\right)^2.$$

Thus total cartel member cost:

$$C_c = n\alpha\frac{1}{2}\left(\frac{Q_c}{n\alpha}\right)^2 = \frac{1}{2}\frac{Q_c^2}{\alpha n}.$$

Thus total profits for the cartel,

$$\Pi_c = pQ_c - \frac{1}{2}\frac{Q_c^2}{\alpha n}.$$

$$\Pi_c = pn[1-(2-\alpha)p] - \frac{1}{2\alpha n}n^2[1-(2-\alpha)p]^2.$$

$$\Pi_c = pn[1-(2-\alpha)p] - \frac{1}{2\alpha}n[1-(2-\alpha)p]^2.$$

Since the cartel chooses price,

$$\frac{d\Pi_c}{dp} = 0 \Longrightarrow \frac{d}{dp}[pn - 2p^2n + \alpha p^2 n - \frac{1}{2}\alpha^{-1}n(1-2p+p\alpha)^2] = 0.$$

$$\frac{d\Pi_c}{dp} = 0 \Longrightarrow n - 4np + 2n\alpha p - \alpha^{-1}n(1-2p+\alpha p) * (-2+\alpha) = 0.$$

$$n - 4np + 2n\alpha p - \alpha^{-1}(\alpha n - 2n)(1-2p+\alpha p) = 0.$$

$$n - 4np + 2n\alpha p - \alpha^{-1}[\alpha n - 2\alpha np + \alpha^2 np - 2n + 4np - 2n\alpha p] = 0.$$

$$1 - 4p + 2\alpha p - 1 + 2p - \alpha p + 2\alpha^{-1} - 4p\alpha^{-1} + 2p = 0.$$

$$\alpha p + 2\alpha^{-1} - 4p\alpha^{-1} = 0.$$

$$\alpha^2 p + 2 - 4p = 0.$$

$$p(\alpha^2 - 4) + 2 = 0 \Longrightarrow p = \frac{2}{4-\alpha^2}.$$

Inserting p back into the profit functions of the cartel and the fringe firms,

$$\pi_f = \frac{1}{2}p^2 = \frac{1}{2}\left[\frac{2}{4-\alpha^2}\right]^2 = \frac{2}{(4-\alpha^2)^2}.$$

$$\pi_c = \frac{\Pi_c}{\alpha n} = \frac{pn[1-(2-\alpha)p] - \frac{1}{2\alpha}n[1-(2-\alpha)p]^2}{\alpha n}.$$

$$= \frac{pn - 2p^2n + \alpha p^2 n - 1/2\alpha^{-1}[n - 4np + 2\alpha pn + 4p^2n - 4\alpha p^2 n + \alpha^2 p^2 n]}{\alpha n}.$$

$$= \frac{p^2\alpha n/(2) - n/(2\alpha) + 2np/(\alpha) - 2p^2 n/(\alpha)}{\alpha n}.$$

$$= \frac{\left[\frac{2}{4-\alpha^2}\right]^2 \alpha/2 - 1/(2\alpha) + 2\left[\frac{2}{4-\alpha^2}\right]/\alpha - 2\left[\frac{2}{4-\alpha^2}\right]^2/\alpha}{\alpha}.$$

$$\pi_c = \frac{1}{2}\frac{1}{4-\alpha^2}.$$

- Proposition 1: Since the fringe firms set their profit maximizing quantity unlimited by the cartel's rules, $\pi_f > \pi_c$.

- Proposition 2: We would expect $\frac{d}{d\alpha}[\pi_c(\alpha)] > 0$. Thus larger cartels hold up profits more stably.

- A main result is that there will be some number of cartel members greater than 1 which will form a stable cartel in the sense that no one wants to leave (interal stability) and no one wants to join (external stability.)

- Consider the following illustration for the case of $n = 4$ given the profit functions above,

α(fraction in cartel)	αn (# cartel firms)	π_f	π_c
$\frac{1}{2}$	2	$\frac{2}{(3\frac{3}{4})^2} = 0.142$	$\frac{1}{2}\frac{1}{3\frac{3}{4}} = 0.133$
$\frac{3}{4}$	3	$\frac{2}{(3\frac{7}{16})^2} = 0.169$	$\frac{1}{2}\frac{1}{3\frac{7}{16}} = 0.145$
1	4	$\frac{2}{3^2} = 0.222$	$\frac{1}{2}\frac{1}{3} = 0.167$

- So only a cartel of size 3 is stable. Consider one of size 4 and all firms making profits $= 0.167$. A firm inside the cartel will look and see that if he leaves the cartle, he will make 0.169 (the profits for a fringe firm given that there are only 3 firms in the cartel). Hence, he will leave. When there are 3 firms in the cartel there are no incentives to leave because $0.145 > 0.142$. Also, when there are only 3 firms in the cartel, the fourth firm on the fringe has no incentive to join since $0.169 > 0.167$. Hence a cartel of size 3 is stable and satisfies the above propositions.

- The problem with this model is that we have assumed firm entry is not possible. If this were the case, the cartel would eventually fall apart and as $n \longrightarrow \infty$, $p \longrightarrow MC$.

4.2 Incomplete Information Model (Green-Porter)

- Where there is incomplete information in the market, firms can cheat the cartel and perform secret price cuts. If cheating was detectable, we would be back in the situation of the "Folk Theorem" from before. In this case however, cheating is difficult to detect.

- Consider a dynamic game based on the simple cournot model. All information is common knowledge except the firm's "own output." Firms face a downward sloping demand curve. If a firm deviates, though their quantity is not necessarily detectable, if one firm cheats and increases production, the other firms will know because the market price will fall. Thus it would be relatively basic to set up a trigger strategy to avoid this and the cartel would be stable. However, now introduce a stochastic demand schedule.

$$p_t = p(Q_t) * \theta_t.$$

Where Q_t is total industry output at time t and θ is a multiplicative shock that is *iid* with C.D.F. $F(\theta)$.

- Hence the appearance of lower prices in the market may be either the sign of a cheating member of the cartel or simply a change in market demand. It is difficult to tell which.

- There are n firms in the industry producing a homogeneous product with profit per period:

$$\pi_i(q_i, p) \; \forall \; i.$$

- A strategy takes the following form:

$$q_{it} = \begin{cases} y_i & \text{if } t \text{ is ``normal''} \\ z_i & \text{if } t \text{ is ``reversionary''} \end{cases} \tag{1}$$

  Firms use the current market price as a trigger to switch between strategies.

- If p falls below some trigger price, $\tilde{p}$, then firms switch to reversionary action, (cournot behavior), for T periods.

- An equilibrium is a triple: $\{q^*, \tilde{p}, T\}$.

- Question is: does there exist a triple that supports a high price (monopoly price for example) such that given this triple, the strategies in which each firm sets q^* in all periods is a NE? Next week...

5 Week 5

- More on the Green Porter model. Define, $V_i(q_i)$ as the NPV of the profit stream of firm i as of some "normal period" given that its strategy is to set output at q_i in normal periods.

$$V_i(q_i) = \underbrace{\pi_i(q_i)}_{\text{Normal profits}} + \underbrace{\delta(1-\alpha(q_i))V_i(q_i)}_{\text{No reversion Profits}}$$

$$+ \underbrace{\alpha(q_i)}_{\text{P(Deviation)}} \left[\underbrace{\sum_{t=1}^{T-1} \delta^t \pi_i(q_c)}_{\text{Reversion Profits}} + \underbrace{\delta^T V_i(q_i)}_{\text{Further Normal Profits}} \right].$$

Where q_i is production in a normal period and q_c is the quantity produced in a reversionary period, which will be assumed to be a cournot game.

- Solving for $V_i(q_i)$,

$$V_i(q_i) = \frac{\pi_i(q_i) + \alpha(q_i)\sum_{t=1}^{T-1}\delta^t \pi_i(q_c)}{1 - \delta(1-\alpha(q_i)) - \delta^T \alpha(q_i)}.$$

- Note that $\sum_{t=1}^{T-1}\delta^t = \sum_{t=1}^{\infty}\delta^t - \sum_{t=T}^{\infty}\delta^t = \delta(1+\delta+\delta^2+\delta^3+\cdots) - \delta^T(1+\delta+\delta^2+\delta^3+\cdots) = \delta(\frac{1}{1-\delta}) - \delta^T(\frac{1}{1-\delta}) = \frac{\delta-\delta^T}{1-\delta}$.

- Thus,

$$V_i(q_i) = \frac{\pi_i(q_i) + \alpha(q_i)\frac{\delta-\delta^T}{1-\delta}\pi_i(q_c)}{1-\delta+(\delta-\delta^T)\alpha(q_i)}.$$

- Multiplying top and bottom by $(1-\delta)$ and writing $\pi_i(q_i) = \pi_i(q_c) + (\pi_i(q_i) - \pi_i(q_c))$,

$$V_i(q_i) = \frac{[(1-\delta)+\alpha(q_i)(\delta-\delta^T)]\pi_i(q_c) + (1-\delta)(\pi_i(q_i)-\pi_i(q_c))}{(1-\delta+(\delta-\delta^T)\alpha(q_i))(1-\delta)}.$$

Thus, for clarity,

$$V_i(q_i) = \frac{\pi_i(q_c)}{1-\delta} + \frac{(1-\delta)(\pi_i(q_i)-\pi_i(q_c))}{((1-\delta)+(\delta-\delta^T)\alpha(q_i))(1-\delta)}.$$

$$V_i(q_i) = \frac{\pi_i(q_c)}{1-\delta} + \frac{(\pi_i(q_i)-\pi_i(q_c))}{(1-\delta)+(\delta-\delta^T)\alpha(q_i)}.$$

- Now we would like to examine the effects of changing firm i's quantity on the NPV of his profit stream. Thus, we need to find, $\frac{\partial V_i}{\partial q_i}$. Thus,

$$\frac{\partial V_i}{\partial q_i} = 0 \Rightarrow [1-\delta+(\delta-\delta^T)\alpha(q_i)]\frac{d}{dq_i}\pi_i(q_i) = (\delta-\delta^T)\{\frac{d}{dq_i}\alpha(q_i)\}[\pi(q_i)-\pi(q_c)].$$

- Or rewritten,

$$\underbrace{\frac{d}{dq_i}\pi_i(q_i)}_{\text{Gains from Deviation}} = \underbrace{\frac{(\delta-\delta^T)[\pi(q_i)-\pi(q_c)]}{[1-\delta+(\delta-\delta^T)\alpha(q_i)]}\{\frac{d}{dq_i}\alpha(q_i)\}}_{\text{Expected loses}}.$$

  The idea is that once you deviate, you increase the probability of a reversionary period so you have to equate the expected gains to how much the firm could lose in a period of cournot competition.

- It is shown (in the article) that there does exist triples that satisfy these equations. If we assume that all firms know the system and understand that deviating is not profitable, if there is ever a fall in price, everyone knows that it must be the result of low demand and not of a deviation from the cartel. However, once this sort of attitude is instilled, there is the optimal strategy to deviate because no one will think that you're deviating, but rather it is just demand falling. But if this spreads the cartel will collapse anyway so for this reason, the cartel is stable. The key result is that of a short sharp shock. So T is small but the loses from deviating are large. It is also important that the cartel will only break down if the demand shock is unanticipated. If everyone knows that a slow down is coming for example, there will be no need to let the trigger set in and the price fall. Firms could actually cut back on production even more during anticipated demand slowdowns.

5.1 Background on Predatory Pricing

- Firms are facing a new entrant into an industry. As soon as the entrant comes in, the incumbents cut prices so that everyone is making losses and the entrant get discouraged by the low prices and low profits and decides to leave. Then the incumbents reinstate the higher and more profitable prices. Bastards.

- Example: shipping cartels and a "Phantom Ship." Suppose there is a shipping industry that ships goods on a circular route in the Atlantic. If a new firm sees that current firms are making profits and enters this industry, one way to drive him back out is by this phantom boat. When the new entrant goes into a port, he makes an offer to a company and loads up his cargo. After it is all loaded, the phantom boat comes along and says that he will beat the new guy's price. Since the entrant's boat is already loaded, he decides to lower his price instead of transfering the cargo. Thus the entrant goes to the next port and the phantom boat follows. The idea is that the phantom boat never carries any cargo but just creates the threat of possibly carrying cargo at a lower price. The entrant carries all the cargo, but makes no profits. Thus the entrant is discouraged and leaves the market.

- The problem with predatory pricing is how one detects the difference between predation and just normal competition.

- We will soon discuss two types of models: 1) Long Purse: the incumbent has more money and can take loses for a longer period than the entrant. So immediately after cutting prices, the firms compare finances and the entrant realizes that he cannot continue in this market at such low prices. Thus, immediately, he exists. 2) Chain store paradox (Reputation): There is no long purse in this case but the idea is that the incumbent will drive out the first firm to try to enter with low prices and this will discourage others from doing so in the future therefore creating a reputation for predatory pricing.

6 Week 6

6.1 Kreps - Wilson Model: Chain Store Model

- The chain store model involves firms relying on a reputation effect for keeping entrants out of a market.

- Consider a constituent game, G, such that the first move is made by the entrant: does he enter or stay out. If he stays out, the payoffs for the entrant and monopolist are $(0, a)$ respectively with $a > 1$. If the entrant enters, the monopolist can fight the entrant in a price war and thus payoffs are $(b - 1, -1)$ with $0 < b < 1$. Or the monopolist can acquiesce and payoffs are $(b, 0)$. **[G-6.1]**

- This type of game parallels Selten's Paradox where we have a finite number of periods and the result is a SPNE where the entrant enters and the incumbent acquiesces in every period. This is simple enough to see from the game above by backward induction.

- However, what happens if the monopolist fights initially and moves off the equilibrium path? This may sound irrational if both the monopolist and entrant have perfect information about payoffs. But if we introduce a little imperfect information, (in this case about the monopolist's payoffs), then we'll get possibly different payoffs with some positive probability which would drive the monopolist to fight entrants off with a price war.

- Consider the game above, G, as the situation where the monopolist is weak. That is if the entrant enters, the monopolist will acquiesce and become a duopolist. Now define a new game, G', with the following payoffs: No entry: $(0, a)$ as before. **[G-6.2]** Entry and Fight: $(b - 1, 0)$. Entry and Acquiesce $(b, -1)$. In this game the monopolist is strong in that if the entrant enters, the monopolist will fight. Thus the SPNE is for the entrant not to enter.

- This can be thought of as an initial move made by nature such that,

$$\text{With Probability } 1 - \delta\text{, Play } G \;\Rightarrow\; \text{Weak Monopolist.}$$

$$\text{With Probability } \delta\text{, Play } G' \;\Rightarrow\; \text{Strong Monopolist.}$$

- Formal Analysis of the Game. $N+1$ players where the $N+1^{st}$ player is the monopolist and there are N potential entrants. G or G' is played by the monopolist against each entrant in turn. Total payoff is the sum of payoffs in G.

- A behavior strategy is a rule specifying the action to take at each node.

- We must now define the concept of Sequential Equilibrium: Equip a player with a set of beliefs (or assessments) such that:

  - a) Whenever a player chooses an action, he has some assessment of which node he is at.

- b) These assessments obey bayes law where ever it applies (ie, when ever there are positive prior probabilities).
- c) From each information set (not each node) the strategy used is optimal given the strategies of opponents and the assessments.

- NOTE: the labelling of the nodes goes backwards in time, so the monopolist plays the game against player N, then $N-1$, ..., then 2, then 1.

- In our present context, an assessment is an updated probability that a monopolist is strong, that is, a function P_n, mapping the history of past moves, h_n, into a probability.

$$P_n : h_n \longmapsto Prob.$$

- Since this problem is rather difficult to solve (guess and check), we now state what constitutes a sequential equilibrium which is just one of possibly many. Proof will come later.

- Assessments: $P_n(h_n)$ is defined recursively. Initially let, $P_n = \delta$. So in the first stage, the probability that the monopolist is strong is δ. After the first period,

  - If no entry occurs or if $P_{n+1} = 0$, then $P_n = P_{n+1}$. Note that P_{n+1} comes before P_n. Basically this means that if the monopolist has ever shown a weakness in the past (he has acquiesced), he can never regain his reputation as being strong.
  - If entry occurs and $P_{n+1} > 0$, then,
    - If the monopolist fights, $P_n = max\{b^n, P_{n+1}\}$. Note this covers the case where δ is so high that $P_{n+1} = P_n = \delta > b^n$. Thus $Prob(fight) = 1$ and updating becomes $P_n = P_{n+1}$.
    - If the monopolist acquiesces, $P_n = 0$.

- The Monopolist's Strategy:

  - If the monopolist is strong, always fight.
  - If the monopolist is weak and entry occurs, then:
    - If $N = 1$, acquiesce since this is the last stage of the game and he will do better by acquiescing.
    - If $N > 1$ and $P_n \geq b^{n-1}$, fight.
    - If $N > 1$ and $P_n < b^{n-1}$, fight with probability $\dfrac{(1-b^{n-1})P_n}{(1-P_n)b^{n-1}}$.

- The Entrant's Strategy:

  - If $P_n > b^n$, Don't Enter.
  - If $P_n = b^n$, Enter with probability, $(1 - \frac{1}{a})$.
  - If $P_n < b^n$, Enter.

- Proof:
  - Step 1. Make sure the assessments satisfy Bayes Law. As cases are trivial except that in which a weak monopolist fights. Bayes law then requires,

$$
\begin{aligned}
P_{n-1} &= P(\text{Strong Monopolist} \mid \text{Fight}). \\
&= \frac{P(\text{Strong Monopolist AND Fight})}{P(\text{Fight})}. \\
&= \frac{P(\text{Fight} \mid \text{Strong}) * P(\text{Strong})}{P(\text{Fight} \mid \text{Strong}) * P(\text{Strong}) + P(\text{Fight} \mid \text{Weak}) * P(\text{Weak})}. \\
&= \frac{1 * P_n}{1 * P_n + \frac{(1-b^{n-1})P_n}{(1-P_n)b^{n-1}}(1-P_n)}. \\
&= \frac{P_n}{P_n + \frac{(1-b^{n-1})P_n}{b^{n-1}}}. \\
&= \frac{1}{1 + \frac{1-b^{n-1}}{b^{n-1}}}. \\
P_{n-1} &= b^{n-1} \\
P_n &= b^n
\end{aligned}
$$

  - Steps 2 & 3: Next week.

7 Week 7

7.1 Continued Analysis of Krep/Wilson Model

- Recall Monopolist's Strategies: If the monopolist is weak:
  - If $n = 1$, then acquiesce.
  - If $n > 1$, and $P_n \geq b^{n-1}$, fight.
  - if $n > 1$, and $P_n < b^{n-1}$, fight with probability $\dfrac{(1-b^{n-1})P_n}{(1-P_n)b^{n-1}}$.
- **[G-7.1]** Graphically, the increasing b^n line crosses the δ line at one point. Up until that point, the monopolist is going to act strong so no entrant would consider coming in. But after the intersection, the monopolist fights with a certain probability. The entrant also enters with a certain probability. Along the equilibrium path, the monopolist will be indifferent between fighting and acquiescing and the entrant will be indifferent between entering and staying out.
- Entrant's Strategy (Part two of proof from last week).
  - Recall the game tree from the game, G. The entrants payoffs were 0 if he stayed out, b if he entered against a weak monopolist, and $b-1$ if he entered against a strong monopolist. Thus if the probability that the monopolist fights is equal to b, then the entrant's expected payoff if he enters is:

$$b(1-b) + (b-1)b = b - b^2 + b^2 - b = 0.$$

  - We focus attention on the case where we are on the path at the end of the game where $P_n = b^n$. **[G-7.2]** Graphically this is b^n line above δ. In this area, the entrant enters with probability $1 - \frac{1}{a}$. To establish the result, we need to show that the entrant is indifferent when $P_n = b^n$.
    Proof: Consider the probability that the monopolist is going to fight upon entry:

$$P(fight) = P(fight|strong) * P(strong) + P(fight|weak) * P(weak).$$

$$P(fight) = 1 * P_n + \frac{(1-b^{n-1})P_n}{(1-P_n)b^{n-1}}(1-P_n).$$

    Now let $P_n = b^n$ because we are interested in the equilibrium path. Thus,

$$P(fight) = b^n + \frac{(1-b^{n-1})b^n}{(1-b^n)b^{n-1}}(1-b^n).$$

$$P(fight) = b^n + \frac{(1-b^{n-1})b^n}{b^{n-1}}.$$

$$P(fight) = \frac{b^n b^{n-1} + (1-b^{n-1})b^n}{b^{n-1}}.$$

$$P(fight) = \frac{b^n}{b^{n-1}} = b.$$

- So the $Prob(fight) = b$ as required. Thus the reasoning goes as follows: If the probability that the monopolist is strong is b^n then the probability that he will fight is equal to b. If the probability of fighting is b, then the entrant's expected payoff is 0 so he is indifferent between entering and staying out.

- Monopolist's Strategy (Step 3 of Proof)
  - Need to show the optimality of the strategy of a weak monopolist that is optimal at each stage. We proceed by way of a backward induction from the final period.
  - Consider the final period, stage 1. Here the strategy prescribes that the weak monopolist acquiesces. It is clear from G that this is optimal. The weak monopolist's payoff is -1 if he has to fight off an entrant while if he acquiesces, he gets either 0 if entry occurs, or $a > 1$ is no entry occurs. So what is the expected payoff in the last period? We have estabilished already that $P_n = b^n = b$. Therefore the entrant is indifferent between entry and staying out. (expected payoff 0). Thus, the entrant enters with probability $1 - \frac{1}{a}$. So the weak monopolist's expected payoff is:
  $$(1 - \frac{1}{a}) * 0 + \frac{1}{a} * a = 1.$$
  - In the second to last period, stage 2, the key point to examine is what is the value of retaining my reputation by fighting if an entrant arrives in period 2? The monopolist is using a mixed strategy, so we need to show that the payoff from fighting is equal to the payoff from acquiescing. If he faces entry and fights in period 2, the expected total payoff is the sum of this period payoff: -1 and the expected payoff next period, 1. So the total payoff is 0. This shows,
  $$Payoff(Fight) = Payoff(Acquiesce) = 0.$$
  - If the monopolist does not face entry in period 2, his payoff is a. Given this however, the monopolist's next period assessment (stage 1) is $P_n = b^2$ (Recall from assessments). Since,
  $$b^2 < b^1 = b,$$
  entry will occur in the last period and the monopolist with acquiesce and get 0. So the total expected payoff in periods 2 and 1 is equal to:
  $$\underbrace{(1 - \frac{1}{a})}_{Probabilty\ of\ Entry} (0 + 0) + \underbrace{\frac{1}{a}}_{Probability\ of\ no\ entry} (a + 0) = 1.$$
  - So monopolist's expected payoff along the equilibrium path, when assessments are following the path $P_n = b^n$, is 1.
  - This show optimality up to period 2, continuing this induction argument, we show optimality along the final part of the path.

- Intuitively, in the early stages, the monopolist has a lot to lose later on by not fighting so fighting early is optimal and then fight later with a certain probability banking on the reputation created in the early stages.
- Thus Reputations can be a substitute for imperfect information in a market.
- Summing up on Predation: There are only two good models of predation as equilibrium outcomes. The Reputation model (Kreps/Wilson) and the Long Purse Argument. The long purse rests on a simple backward induction. A monopolist can afford to finance losses for n_1 periods, while the entrant can only finance losses for $n_2 < n_1$ periods.
- It's still very difficult to tell a predation story because these two models are difficult to apply to individual cases. (ReaLemon)
- The only legal criteria available is the Arreda-Turner Rule which says that if $P < AVC$, this is evidence of predation. However under this rule and other similar ones, there is always the chance of making type I and II errors.

8 Week 8

8.1 Product Differentiation

- Two types of product differentation: horizontal and vertical. The following analysis is of horizontal and next week we will discuss vertical.

- Consider's Hotelling's Model (1929): Ice Cream stands arranged along a beach so people choose to go to the ice cream stand that is closest to them. This is a sort of spacial horizontal product differentiation where the products are identical but their spacing makes them different.

- Define consumer's utility as:

$$u = Constant - p - t * d,$$

  where d is the distance traveled to reach the stand.

- The key idea of horizontal price differentiation is that if all prices are equal across producers, some consumers will choose to buy from firm 1, others from firm 2, etc.

- Define the Marginal Man: the guy right in the middle of two producers (ice cream stands).

- We can also consider a variant of the model called the Circular Road Model: N firms arranged equally spaced around a circle of circumference 1. Assume firms have cost 0.

- Thus, unrolling the circle, we are back at the beach type of idea but we don't have to worry about firms on the "ends" of the beach because you can think of the circle idea to get "around" it. :)

- We need a Nash Equilibrium in prices. First explore the case where all prices are equal which is the obvious candidate for equilibrium. Let $\bar{p}$ be the equilibrium price and consider a deviant firm who sets p_i. Let d be the distance from the marginal man to the firms on either side of him. Clearly if the distance between any two firms is $\frac{1}{N}$ and the marginal man is d units away from one firm, he is $\frac{1}{N} - d$ units away from the other firm. The equation of the marginal man in between a conformist and the deviant that will make him indifferent between the two is:

$$p + td = \bar{p} + t(\frac{1}{N} - d).$$

  Solving,

$$2td = \frac{t}{N} - (p - \bar{p}).$$

  Firm sales are equal to one half of the area to the left of him and one half the area to the right. Thus,

$$Sales = 2d.$$

Substituting in,

$$Sales \equiv x = 2d = \frac{1}{N} - \frac{(p - \bar{p})}{t}.$$

Therefore firm profits are:

$$Profits \equiv \pi = px = p\Big[\frac{1}{N} - \frac{(p - \bar{p})}{t}\Big].$$

Or rewritten,

$$\pi = \frac{p}{N} - \frac{p^2}{t} + \frac{p\bar{p}}{t}.$$

The optimal choice of firm i is found my maximizing profits with respect to price. Thus,

$$\frac{\partial \pi}{\partial p} = \frac{1}{N} - \frac{2p}{t} + \frac{\bar{p}}{t}.$$

For a symmetric NE with equal prices, set $p = \bar{p}$, thus,

$$\frac{\partial \pi}{\partial p} = \frac{1}{N} - \frac{2\bar{p}}{t} + \frac{\bar{p}}{t}.$$

$$= \frac{1}{N} - \frac{\bar{p}}{t} = 0.$$

Thus,

$$\bar{p} = \frac{t}{N}.$$

- Thus consider two limiting cases. If $t \longrightarrow 0$, or consumers do not care about the distance they have to travel, then we get homogeneous products in terms of spacial distance. So $\bar{p} = 0$ as in a Bertrand model of $P = MC = 0$.

- The second limiting case is when $N \longrightarrow \infty$. Thus there are many many firms so each producer has a neighbor which produces a close substitute to his good. So again, $\bar{p} = 0$.

8.2 Some Generic Properties of Product Differentation

- 1) We can find sets of locations along the line such that we have any number of firms, N, each with a positive market share at equilibrium. And if the density of consumers on the line is sufficiently large, (we set it equal to unity above), then the profits of each firm can be made large enough to cover some fixed entry cost. Thus you can have a very fragmented market equilibrium with a lot of small firms.

- 2) We can analyze the choice of location by looking at a 2 stage game where firms enter and choose locations in stage 1 and then compete in prices in stage 2. Details of this will come next term. We can then ask the question of given a certain market set up, is it profitable for another firm to enter in between two firms. We could construct examples where all firms in equilibrium make making strictly positive profits (Super Normal Profits) but it is unprofitable for a new firm to enter because they will only get a smaller slice of the pie.

- 3) One final idea: multiple equilibrium is endemic in these types of models. Consider a set up where 6 different firms are spaced equally along the beach. Suppose now that firm 1 buys out firms 3 and 5 and firm 2 buys out firms 4 and 6. The market setup is exactly the same with each firm having competitors on each side of him and splitting the market in between them. In this case there are only two firms in the market, but both scenerios are an equilibrium.

8.3 An Application of Vertical Restraints

- Matthewson - Winter Article.
- Consider a setup with two firms: an upsteam firm, a manufacturer, and a downstream firm, the retailer. The manufacturer sells to retailer which then sells to consumers. The manufacturer is controlling the retailer's price by its wholesale price. The problem is that the profit maximizing prices for each firm will be different. Is it because the manufacturer is monopolizing the market or is it because the retailer is not taking into account some externality (the manufacturer's profits).
- The retail firms are represented on a circular road with uniform density of consumers equal to v. v could be considered the size of the market.
- The manufacturer supplies the retailer at the wholesale price, p_w. The retailer sells to consumers at price, p.
- There is a fixed cost, F, for a retailer to establish a retail outlet.
- Advertising is done by the retailer and it conveys information as to the availability of the product. The cost of advertising at density, A, over an interval of the line of length ds is $b * A * ds$ where b is the unit cost of advertising.
- If the density of advertising is A, then the fraction of consumers who become informed as to the availabilty of the product is denoted $h(A)$ with $h' > 0$ and $h'' < 0$.
- So the density of informed consumers at any point is $v * h(A)$.
- Consumers, once informed, have individual demand schedules, $x = f(p + ts)$ where t is unit transport cost and s is the distance traveled.
- Finally we introduce the idea of advertising spillovers, or externalities that one retailer gets from advertisments from another. We denote α as the fraction of advertising messages that go outside the advertisers market area.
- We capture this by writing the fraction of informed consumers for firm i not as $h(Ai)$, but as,

  $$h(\alpha A + (1 - \alpha)A_i),$$

  where A is the market average of advertising and A_i is firm i's advertising efforts.

- The formalities of the model: Define the market radius as R_i. Let $\frac{1}{N} = X$. Therefore the marginal man is R_i from the deviant producer and $X - R_i$ from the conformist producer. Setting equal as before:

$$p_i + tR_i = p + t(X - R_i).$$

Solving for R_i,

$$R_i = \frac{p - p_i}{2t} + \frac{X}{2}.$$

The Sales for firm i:

$$2vh(\alpha A + (1 - \alpha)A_i) \int_0^{R_i} f(p_i + ts)ds.$$

The retailer's profits including fixed fees and advertising expenses,

$$\pi^R(p_i, A_i) = 2\Big[vh(\alpha A + (1 - \alpha)A_i)(p_i - p_w) \int_0^{R_i} f(p_i + ts)ds - R_i bA_i\Big] - F.$$

And optimizing with respect to advertising,

$$\frac{\partial \pi^R}{A_i} = 2vh'(A)(1 - \alpha)(p - p_w) \int_0^{R} f(p + ts)ds - 2Rb = 0.$$

Similiarly for the monopolist (manufacturer),

$$R\frac{\partial \pi^M}{A} = vh'(A)(p - c) \int_0^{R} f(p + ts)ds - Rb = 0.$$

- Thus the profit maximizing decisions are different because of p_w compared with c. If there exists externalities, $\alpha > 0$, then the effects are worsened.
- There are 3 causes of these externalities that can arise here:
  - $p_w > c$ ("Vertical Externality" or "Double Marginalization")
  - Price competition between retailers which cause the price, p, to be too low.
  - $\alpha > 0$: Advertising spillovers.
- How do we get around these externalities?
  - There are various packages of constraints. If $\alpha = 0$, we can use a franchaise fee plus a closed territory distribution which eliminates externality number 2 because it eliminates price competition. The franchaise fee extracts the extra profit.
  - Equivalently, we can use the package of a franchaise fee plus forcing (a minimum quantity that must be sold).
  - Another method is to have a franchaise fee plus setting $p_w = c$ which eliminates externality number 2. Combine with a minimum retail price to fix retail price at the monopolists's first best level (?)

9 Week 9

9.1 Vertical Product Differentiation

- Consider two firms, A and B, who sell at the same price, $P_A = P_B = P$. In horizontal product differentiation (HPD), some consumers prefer A and some prefer B, but in vertical product differentiation (VPD), all consumers choose the same product, namely the higher quality item.

- Note that it is consumer's "perceived" quality that matters. Thus it is consumer's willingness to pay that will determine A and B's market shares.

- The model goes as follows: Assume a number of firms offer distinct, substitute goods which vary in quality. (Note that if the goods were homogeneous, this would just collapse to a Bertrand game.)

- Consumers buy one unit or none.

- Firms have zero cost.

- Goods labeled $k = 1 ... n$ and thus firm k sells product k at price p_k.

- Consumer's income levels are distributed uniformly over the interval $t = [a, b]$.

- Consumer's Utility level:

$$u(t, k) = u_k(t - p_k)$$

$$u(t, 0) = u_0 t.$$

  Where t is the consumer's income level and,

$$0 < u_0 \text{ constant } < u_1 < \cdots < u_k < \cdots < u_n.$$

- Obviously, the consumer's income level is what determines his willingness to pay and thus the demand for each good.

- $u(t, 0)$ is what the consumer will get if he does not buy any of the n goods and spends his income and some other bundle of goods. This bundle is referred to as the "Hicksian Composite Commodity" and is defined by holding all relative prices of goods in other markets constant.

- Note that both utility functions are linear depending on whether or not the consumer chooses to buy the good. Refer to graph in the notes for this next part. **[G-9.1]** Consider an individual who chooses not to purchase any of the goods. Graphically, if we take the utility of the consumer at his income level when he does NOT buy any of the goods and then subtract p_k off his income level, the utility at this new level $u(t - p_k)$, must be smaller than the utility at the original level.

- Consider "Marginal Man." He is just indifferent between buying good k and good $k-1$ which is a bit less expensive but also of lower quality. Consider graph in notes that shows that the price gap for the marginal man increases as his income rises. **[G-9.1]** Thus the more wealthy people are more willing to pay more for quality. Thus,

$$u_k(t_k - p_k) = u_{k-1}(t_k - p_{k-1}).$$

Where t_k is the marginal man's income level.

- Solving the marginal man's equation for t_k,

$$u_k t_k - u_{k-1} t_k = u_k p_k - u_{k-1} p_{k-1}.$$

$$t_k(u_k - u_{k-1}) = u_k p_k - u_{k-1} p_{k-1}.$$

$$t_k = \frac{u_k p_k - u_{k-1} p_{k-1}}{u_k - u_{k-1}}.$$

- Now define the C_k as:

$$C_k = \frac{u_k}{u_k - u_{k-1}}.$$

This quantity will show up in the algebra below so it will simplify things. Note that $C_k > 1$ because $u_{k-1} < u_k$. Also, as the quality of these two goods get very close, $C_k \to \infty$.

- Thus we can simplify the above equation to,

$$t_k = C_k p_k + (1 - C_k) p_{k-1}.$$

- FOCs:

$$\frac{\partial t_k}{\partial p_k} = C_k > 1.$$

$$\frac{\partial t_k}{\partial p_{k-1}} = 1 - C_k < 0.$$

- Now consider the highest quality good, good n. Let $t_n > a$ so that more than one good survives the market. (We avoid a corner solution in this case). Note that the top quality producing firm can always survive because he can always price the lowest and gain the whole market. Define the profits of the top quality firm:

$$\pi_n = R_n = p_n(b - t_n).$$

Because costs are zero, profits equal revenues. p_n is the price charged and $b - t_n$ is the fraction of the market that the top quality firm gets. Taking the derivative,

$$\frac{\partial \pi_n}{\partial p_n} = b - t_n - p_n \frac{\partial t_n}{\partial p_n}.$$

Substituting in from above,

$$\frac{\partial \pi_n}{\partial p_n} \Rightarrow b - t_n - p_n C_n = 0.$$

But also from above,

$$p_n C_n = t_n - p_{n-1}(1 - C_n).$$

Substituting in,

$$b - t_n - t_n + p_{n-1}(1 - C_n) = 0.$$

$$b - 2t_n + p_{n-1}(1 - C_n) = 0.$$

$$b - 2t_n - p_{n-1}(C_n - 1) = 0.$$

Since $C_n - 1 > 0$,

$$b - 2t_n > 0.$$

Or,

$$t_n < \frac{1}{2}b.$$

- Since $a < t_n$, then if $a > \frac{1}{2}b$, it cannot be that more than one firm survives. Thus as long as we are not at a corner solution, there is always a little more room for another firm (lower quality) to enter the market and charge a positive price. Thus the concentration of income becomes important because a more concentrated income base will only allow one quality of good in the market, namely the highest quality product. (See graphs) **[G-9.2]** .

- Now consider the same argument for a general good, k. Profits for firm k,

$$\pi_k = R_k = p_k(t_{k+1} - t_k).$$

FOC:

$$\frac{\partial \pi_k}{\partial p_k} = t_{k+1} - t_k - p_k \frac{\partial t_k}{\partial p_k} + p_k \frac{\partial t_{k+1}}{\partial p_k}.$$

Substituting in from above,

$$\frac{\partial \pi_k}{\partial p_k} \Rightarrow t_{k+1} - t_k - p_k(C_k) + p_k(1 - C_{k+1}) = 0.$$

But also from above,

$$p_k C_k = t_k - p_{k-1}(1 - C_k).$$

Substituting in,

$$t_{k+1} - t_k - (t_k - p_{k-1}(1 - C_k)) + p_k(1 - C_{k+1}) = 0.$$

$$t_{k+1} - 2t_k + p_{k-1}(1 - C_k) + p_k(1 - C_{k+1}) = 0.$$

$$t_{k+1} - 2t_k - p_k(C_{k+1} - 1) - p_{k-1}(C_k - 1) = 0.$$

Since $C_{k+1} - 1 > 0$ and $C_k - 1 > 0$, then $t_{k+1} > 2t_k$, or

$$t_k < \frac{1}{2}t_{k+1}.$$

- It immediately follows therefore:

$$t_n < \frac{1}{2}b, \;\; t_{n-1} < \frac{1}{4}b, \;\; t_{n-2} < \frac{1}{8}b, \;\; t_{n-3} < \frac{1}{16}b, \;\; \ldots$$

- Thus, price competition between the high quality goods drive down their prices to a level such that at equilibrium prices, even the poorest consumer prefers to buy the last surviving good at its equilibrium price rather than the "best non-surviving" good at $p = MC = 0$.

- Result: The Finiteness Property: There exists a bound, B, independent of quality such that at most B firms can have positive sales revenues at a Nash equilibrium in prices. Thus,

$$\text{If } a > \frac{1}{2}b, \text{ then } B = 1.$$

$$\text{If } \frac{1}{2}b > a > \frac{1}{4}b, \text{ then } B = 2.$$

$$\text{If } \frac{1}{4}b > a > \frac{1}{8}b, \text{ then } B = 3.$$

$$\vdots$$

- In HPD, we had the situation where the more consumers that were in the market, the more room there was for another firm to enter the market. In VPD, there is no effect of more consumers in the market. This is an example of the non-convergence principal.

9.2 Cost Structure Extention under VPD

- Suppose that the MC of producing at quality u is denoted by $C(u)$ with $C'(u) > 0$.

- What do we know about the shape of $C(u)$? If the main burden of quality improvements falls on fixed costs, then the shape of $C(u)$ will be fairly flat. Such markets would be computers for example. If the main burden of quality improvments falls on variable costs, then the shape of $C(u)$ will be fairly steep. Such markets would be tables for example.

- Some have just assumed that $C(u)$ is convex, but this would not be a good assumption and the result is to fall back into the hotelling case.

- **[G-9.3]** So now consider a mapping of consumer's preferences (willingness to pay) under the two different cost structures. In the hotelling case, there would be a one to one mapping of consumers to the producers which had their shop closest to them. In this case, if the cost curve is steep, meaning that prices are very different for low versus high quality, we will get the one to one mapping again with low income consumers paying for the lower quality products. However, if the cost curve is flat, as in the case of computers, the price the consumer needs to pay for quality is not very high so even people low on the income scale will be more willing to pay a bit more for the higher quality goods. Thus, the mapping collapses to a fixed point where only the highest quality good survives.

10 Week 10

10.1 Evolutionary Games

- Many ties of the game theoretic setting can be made and specifically in biology, we can model the success of certain species as a sort of game.

- The payoff in this game is the passing on of genes to the next generation. Thus if you “win” the game, you increase your net rate of reproduction.

- Your strategy is your genes. So a certain type of individual plays a given strategy. You really don’t have any choice over your genes.

- We imagine a dynamic under which successful strategies are replicated.

- The idea of finding a mixed strategy Nash equilibrium to this problem is equivalent to finding, in biology, a polymorphic equilibrium. Consider the spider population that is made of aggressive and non-aggressive spiders. For instance when one spider has made a web and another comes along to steal it from him, the spiders play a game where they shake the web and the spider that can shake the web the hardest, wins.

- So we need a standard equilibrium concept which biologists call an “Evolutionarily Stable Strategy” or ESS. Note we are assuming symmetric games so that all players have the same possible strategy set.

- Proceed as follows: We say that “A invades B” if,

  $$V(A|B) > V(B|B).$$

  Thus, when an A meets a B, he does better than when a B meets a B. Thus A begins to dominate over the B’s.

- We say that “B is stable” if,

  $$V(A|B) \leq V(B|B) \; \forall \; A.$$

- Define an ESS as follows. The motivation behind this is that the notion of stability ignores the effect of interaction among A’s (or the mutants in this case). So we must take this into account. Thus B is an ESS if for every mutant, M, EITHER

  $$V(M|B) < V(B|B),$$

  OR

  $$V(M|B) = V(B|B) \; and \; V(M|M) < V(B|M).$$

  The second condition makes an ESS different from a mixed strategy NE.

10.2 Political Science Digression

- Consider Axelrod's Experiments where he considered a finitely repeated prisoners dilemma where according the the book of Nash, the only equilibrium is to defect in every period. He gave this setup to his students and other game theorists and then randomly put the players strategies against each other to find out which would be most successful. He found that a Tit for Tat (TfT) strategy was the "Best." That is to cooperate in the first period and then do exactly what your opponent has done in the previous period for all periods after the first. Clearly if two players are both playing TfT, cooperation will be sustained for the entire game.

- So in a random, evolving game of strategies, the TfT people came out as the dominate part of the population.

- However consider the following theorems which tend to discount Axelrod's findings. First of all, there is no "universally best" strategy, but rather there is an optimal reply strategy to every possible strategy that your opponent chooses. Also TfT is a "collectively stable" strategy for a discount factor δ sufficient high. Note that TfT NEVER wins. Consider putting a TfT against a strategy that always deviates, D. We have the following outcome in a 3 period game:

t	1	2	3
TfT	C	D	D
D	D	D	D

  Thus, in the first round D wins and then in the rest, they tie. Thus D wins.

- However, given that the opponent is playing TfT, the guy playing D is better off playing TfT. Consider,

t	1	2	3
TfT	C	C	C
TfT	C	C	C

  Thus, both players are playing their optimal reply. Thus TfT is collectively stable.

- The set of all players playing TfT is a NE.

- Note also that the always defect strategy, D, is collectively stable as well, but for all discount factors, δ.

- In other words, a population of all "Always Defectors," or D players, cannot be invaded by a TfT in the sense defined above. (It is clear that the TfT would lose every meeting with a D player).

- Axelrod also claimed that virtuous behavior will evolve (ie TfT players). This is clearly not so because as we have shown that an entire population of defectors can never be invaded by a TfT player.

- Thus, the question becomes, how big does a virtuous population have to be to take over? We must introduce the concept of a cluster of "mutants" who form proportion, p, of the population. That is, the probablity that a mutant encounters another mutant is p. How low a value of p can the population of "Always Defects" be invaded by a population of "TfT?"

- Thus Axelrod's theorem called the Basic Characterisation Theorem: The lowest value of p is attained by the class of "maximally discriminating strategies." A maximally discriminating strategy is one that satisfies two conditions:

  - 1) It has the feature that it specifies a play of "Cooperate" even if the rival hasn't yet cooperated.
  - 2) After an initial play of cooperate, it will never again cooperate if playing against an "Always Defect" player and if will always cooperate against someone that is of its own type.