Industrial Organization
Lent Term

Matthew Chesnes
The London School of Economics

March 22, 2002

1 Week 1: 14 Jan - 18 Jan

1.1 Background on Bain Paradigm

- Once again, we refer back to the Bain Paradigm, the idea of how structure determines conduct, conduct determines performance and performance might cause entry into an industry to change the structure and the process starts again. See notes. **[G-1.1]**
- Structure is measured by some concentration measure such as C_k: the concentration of the k largest firms in an industry.
- Conduct is basically if the behavior of the firms in the industry is cooperative or non-cooperative.
- Performance is usually measured by profitability such as return on assets : $\frac{\pi}{A}$.
- If performance is positive, this could induce entry by new firms unless there exists barriers to entry such as advertisting or $R\&D$.
- The list of explanatory variables in this model includes economies of scale, advertising outlays and R and D expenditure. Economies of scale should be an exogenous variable from the point of view of the firm, but advertising and $R\&D$ are both endogenous and should be jointly determined with the structure at equilibrium.
- The Traditional Agenda has been to run two regressions to determine the relationships in the Bain Paradigm.
  - 1) Performance/Profitability: $\frac{\pi}{A} = a + bc_k$. When running this regression, at first the bulk of the papers written found $b > 0$. However, as more and more studies were completed, as is often the case, economists began to question if a relationship existed at all.
  - 2) Structural explanation equation: $C_k = a + b$[Measure of Scale] $+c$[Advertising / Sales] $+d$[R&D / Sales] $+ \ldots$. In running this regression, it is important to distinguish between the concentration/profitability relation and the price/concentration relation. There is an overall tendancy for prices to fall as the the number of firms rises or as the concentration measure, C_k, falls: often plotted as $\frac{1}{C_k}$. See Notes. This regression is also invalid because we have explanatory variables which are endogenous. Thus the regression is misspecified. However for what it's worth, in this regression, economists found $b > 0$ almost always, the coefficient on $\frac{Ads}{y}$ was significant but it should be endogenous. The coefficient on $\frac{R\&D}{y}$ has hardly ever significant.
  - Thus econometricians tried to estimate simultaneous equations, but failed here as well.

1.2 Market Structure

- Consider the following basic structure. We model a 2 or multistage game where in the first stage, all irreversible investments are made or basically all costs are incurred. Then, after some limit point, the point at which, "firm's capability is fixed," all sunk costs have been made. Then in stage 2, the final stage game, we have price competition and profits are determined. See notes. **[G-1.2]**

- To study examples of this, a preliminary distinction must first be made between exogenous and endogenous sunk costs.

  - Exogenous sunk costs are a special sub-case of the large set of endogenous sunk costs. See notes. When sunk costs are exogenous, we mean that all firms must pay a fixed entry fee or setup cost per product produced.
  - The more common endogenous sunk costs are those in which firms choose how much to invest, following entry, in advertising, R&D, etc.

- So consider an example where sunk costs are exogenous first, the special case.

  - Suppose first we have homogenous products. Depending on the type of price competition, the plot of price versus number of firms in the industry will change. Under the bertrand game, the most severe form of price competition, prices drop to MC as soon as more than one firm is in an industry. Cournot competition is slightly less severe and some sort of jointly profit maximizing model results in a constant equilibrium price as new firms enter. See graph in notes. **[G-1.3]** Via backward induction in all these setups, we can determine the equilibrium in the first subgame.
  - Now let S denote the size of the market in the sense that S increases by way of a replication in the population of consumers. (See graph of demand curve pivoting out as S rises). **[G-1.3]** So consider a graph of the concentration of an individual firm on the vertical axis (assuming all firms have equal market share, $C_1 = \dfrac{1}{N}$ gives us the market shares of all firms), with S on the horizontal axis. As S gets larger, the concentration index will depend on the type of price competition again. If we have a bertrand game, since once a single firm is in the industry, a second firm knows that they will both make zero profits if he enters, no additional firms enter. (This is assuming a fixed setup cost, $\epsilon > 0$. Note that in a "Contestability model" one sets $\epsilon = 0$.) **[G-1.3]**
  - In cournot competition, concentration falls as the market grows but relatively gradually and finally in a jointly profit maximizing model, the concentration index falls the fastest in the larger market.
  - So, in general, a rise in the toughness of price competition raises equilibrium concentration for any given size of market. See graph. NB: The phrase, "Toughness of price competition" refers to the functional relationship between concentration (number of firms) and prices (price/cost margins).

- Now consider a second example, the subcase with horizontal product differentiation.
  - What changes in this subcase, is that multiple equilibria appear and the relation between S and C_k now becomes a bounds relationship.
  - Recall the hotelling line example with ice cream vendors lined up on a beach. Suppose we have an equilibrium with firms: $A, B, C, D, \ldots$ all lined up along a beach. We can construct another equilibrium as shown in the notes with higher concentration among firms : $A, B, A, B, \ldots$. Note that S did not change here and we got C_1 to rise. As shown in the graph in the notes, this creates a bound to the relationship between C_1 and S. Anywhere above that bound, we can construct an equilibrium. **[G-1.4]**
  - So the general implication: A characterisation of the $(C_1; S)$ relationship is in general a BOUNDS relationship and not a FUNCTIONAL relationship.

- Now, lets proceed with an endogenous sunk costs example.
  - Consider a "Cournot Model with Quality." Suppose firms can choose a quality level, u at cost $F(u)$. Let all consumers have the same cobb-douglas utility function,

$$U = (ux)^{\delta} Z^{1-\delta}.$$

    Here, u is the quality of the good purchased. x is the quantity of good purchased and δ represents the proportion of income the consumer chooses to spend on the u quality goods. Z represents the outside hicksian composite commodity. Note that δ is independent of relative prices. Denote by S, the total expenditure on the quality good.
  - From the utility function, notice that a consumer will choose a good that offers the best quality/price ratio because of the ux in the utility function. Hence at equilibrium,

$$\frac{p_1}{u_1} = \frac{p_2}{u_2} = \cdots = \frac{p_i}{u_i} = \ldots.$$

  - Suppose all firms offer a common quality, $\bar{u}$, except for one deviant who offers, u. Thus,

$$\frac{u}{p} = \frac{\bar{u}}{\bar{p}} \Longrightarrow p = \frac{u}{\bar{u}}\bar{p}.$$

    Then total expenditure, S is

$$S = \bar{p}Q + \underbrace{\bar{p}\frac{u}{\bar{u}}q}_{\textit{Deviant Good}}.$$

    So the consumers spend $\bar{p}Q$ on all the normal (quality $\bar{u}$) goods and then pay $\bar{p}q$ times this ratio of qualities for the deviant good. Hence solving for the equilibrium price, $\bar{p}$,

$$\bar{p} = \frac{S}{Q + (u/\bar{u})q}.$$

- We can now seek a Nash Equilibrium in quantities (cournot equilibrium), the result is, once we solve for $\bar{p}$ and $\bar{q}$ that equilbrium profit of the deviant firm is:

$$\pi(u|\bar{u}) = S\left\{1 - \frac{1}{\frac{1}{N-1} + \frac{u}{\bar{u}}}\right\}^2.$$

- As can be seen in the graph in the notes, [**G-1.5**] depending on the number of firms in the industry, the final stage profit varies. If the number of firms is small, the gains from a large deviation in quality is also large. As the number of firms get larger, the gains from deviating fall. See week 7 notes of Lent for more information on this derivation.
- The last part of this is not very clear due to how quickly it was covered in lecture. Refer to chapters 2 and 3 of Technology and Market structure. Basically, under the endogenous sunk costs model, if we assume that the cost function is of the following form:

$$F(u) = F_0 u^{\beta},$$

  Then there reaches a point when the size of the market is so large that the concentration index does not fall any more. Therefore, there exists a lower bound to the concentration index. Hence the "Non-convergence theorem." [**G-1.6**]
- Kate's Interpretation: As far as I can tell, as the market size (S) increases the return from spending more (on advertising etc) increases, so the deviant firm moves up the $(F(u), u)$ curve. I'm not sure but I think that this means that the as the spending on advertising increases, then consumers gain greater utility and so I suppose they form strong preferences over the products and as a result, no firms enter the industry (higher fixed costs) and a constant number of firms spend more and more, so that concentration reaches a point where it stops falling - hence the non-convergence theorem.

2 Week 2: 21 Jan - 25 Jan

2.1 Missed Lecture - Kate's Notes

- The central problem that we would now like to look at is the influence of unobservables. Factors that are difficult to measure, proxy, or control for in empirical testing. The two key unobservables in IO: Form of price competition and the nature of the entry process.

- In modelling, we seek characteristics of the industry (observables) and try to see what the outcomes are (observables). However, this really explains nothing about the unobservables. We simply rationalized everything using game theory.

- When searching for the form of price competition, we know that certain relationships define certain forms of price competition.

- In terms of the nature of entry, we may know something about the order in which firms entered the industry (all at once or sequentially). However, we may know this only about a few firms, but not across all industries which would be needed for modeling.

- If we knew both of these things (price competition and method of entry), we would model the sitution as shown in the notes. Note that the type of entry process, via game theory, affects how the firms interact and for example, how much capacity they will bring to the market.

- The idea is that we would like to not work with ONE particular game (since we don't know what it is), but rather a class of possible games. We find a class such that the characteristics that we do know are shared in all the possible models.

- History of Economic thought: 1947 the standard paradigm: set up full model, seek equilibrium and change the parameters to see how the model dynamics work. In modern IO modeling, we go beyond this. The bounds approach determines a NE in state space such that outside of the area cannot be supported by ANY of the models as a NE. See graph in notes.

2.2 The Bounds Approach

- Define an admissable class of models.

- Partition the space of outcomes. Ask yourself, can this outcome be supported as a NE of any admissable model?

- Notation: [**Graph.**] we have an initial stage of the game where firms take actions which involve some costs (R&D etc) and these are irreversible costs. Then beyond some point, "firm's capability becomes fixed." In other words, they have incurred all their sunk costs and are commited to whatever they have set up so far. Beyond this point, we have price competition in one of many different forms which we have studied.

- Define: N_0 as the players in the game. And A represents the space of actions or locations. The set of locations occupied by firm i is denoted $a_i \in A$. A firm in the market is characterised by his $MC = c$, demand schedule, productivity and certain product characteristics. u is a vector of quality variables, one for each research trajectory. The pair of numbers, (c, u), is the firms "capability."

- We want to encompass a wide range of examples by setting up the model in an abstract way.

- Example: 'Capacity Choice Game': The firms choose the level of plant capacity.

- Example: 'Hotelling Model': Firm i chooses location in $[0, 1]$.

- Thus, the "capabilities of the firms" will be defined in different ways depending on the first stage of the game (how it is modeled).

- In an example, A is the set of pairs of numbers, (x, y) with $x \in (0, \infty)$ and $y \in (0, \infty)$.

- In the Capacity choice game, A is the set of real numbers, $y \in (0, \infty)$.

- In the hotelling model, A is the set $[0, 1]$.

- NB: $\tilde{a}_i$ denotes a set of points (bold in the text).

- The outcome is an N_0 - tuple that specifies the $\tilde{a}_i$ for each firm, ie,

$$(\tilde{a}_1, \tilde{a}_2, \tilde{a}_3, \ldots, \tilde{a}_{N_0}).$$

  This is what happens at the penultimate stage of the game when all I's (investments?) have been made.

- It is always possible for a time to choose no points in A, ie Strategy=DO NOT ENTER. This is denoted as $\tilde{a}_i = \{\emptyset\}$. We delete all such firms and denote the number of firms remaining as $N(< N_0)$.

- Now we will define a profit function for each firm, π. This is defined directly over the $\tilde{a}_i$'s as it depends on i's location and other's locations. Thus,

$$Payoff : \pi(\tilde{a}_i|\tilde{a}_{-i}) - \underbrace{F(\tilde{a}_i)}_{Cost\ Function}.$$

  Note that $\tilde{a}_{-i} = (\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_{i-1}, \tilde{a}_{i+1}, \ldots, \tilde{a}_N)$. It is the capabilities of all the other firms.

- The function $F(\cdot)$ is a cost function associated with the capabilities of the firm.

- Note: We consider the class of all stage games of this form: these are games of complete information and we will bias on the pure strategy perfect NE.

- We place 2 restrictions on the game: 2 weak regularity assumptions:
  - 1) There are many firms and the market is big enough to support at least one entrant. There is some ϵ such that for any $\tilde{a}_i \neq \{\emptyset\}$, $F(a_i) \geq \epsilon > 0$. Moreover, N_0 is large enough that the profit, if N_0 firms enter, cannot be so high as to cover the minimum fixed costs, ϵ.
  - 2) Extensive form: Motivating the second restriction: Note that in stage games, if players enter in the sequence $1, 2, 3, \ldots$, then if player i was allowed to delay and enter in position $j > i$, this may raise i's profit. Therefore, we associate with each firm, i, an integer, t_i. Firm i is free to enter any subset of the set of products, A, at any stage t such that $t_i \leq t \leq T$. ie, firm i is restricted to enter the game at a certain time.

- Define what is meant by equilibrium: there are 2 properties that any pure strategy NE must have. Definition. The N-tuple $(\tilde{a}_i)$ is an equilibrium configuration if:
  - 1) Viability or survivor principal: For each firm i,

    $$\pi(\tilde{a}_i|\tilde{a}_{-i}) - F(\tilde{a}_i) \geq 0.$$

    Not strong outside of perfect competition.
  - 2) Stability or the arbitrage principal: There is no set of actions, $\tilde{a}$ such that entry is profitable. For all sets of actions $\tilde{a}$,

    $$\pi(\tilde{a}_{N+1}|\tilde{a}_i) - F(\tilde{a}_{N+1}) \leq 0.$$

    This means that it is not profitable for an additional firm to enter the market. This there are no gaps in the market to be exploited by a firm OUTSIDE the market. See illustration in notes. [**Graph.**]

- We have not specified the form of the entry so we can't talk about "optimal" behavior. ie, we cannot determine whether a firm's location is optimal.

- We are defining an equilibrium concept weaker than the NE as we no longer have the optimality condition. The interesting point here, in relation to agent's rationality, is that it does not require all agents to be optimizers: It merely requires that there should always be available one smart agent who will enter the market and fill the gap.

- The Central Theorem: Inclusion. The set of equilibrium configurations includes all outcomes or configurations that can be supported as pure strategy perfect NE in any game of this class. To do this, we must show that all firms satisfy viability and stability. Viability: if $\pi < 0$, action 'enter' is dominated by 'do not enter' so no NE. Stability, ...

2.2.1 A "Simple" Illustration

- Hotelling simple location game with entry.
- Each firm chooses a point in $[0, 1]$ and the payoff equals the length of the line segment consisting of pairs closest to that firm (with the rule that equal locations share the payoff of that location). See illustration in notes.
- If we fix the number of players, $N = 2$, $N = 3$, $\ldots$ If we have two firms set up in the market and suppose they are located at $\frac{1}{3}$ and $\frac{2}{3}$ along the line. Moving towards the center is profitable for each firm so eventually they will end up right in the center and split the market evenly (though they were already doing this before).
- As we add more players, things get messy.
- Let N_0 firms choose whether to enter at some fixed location or not enter. Let the payoff be given by the length of the line segment you own (market share) times S (market size). As S increases, it becomes profitable for more firms to enter.
- Let the entry fee be equal to ϵ normalized to 1. The necessary condition for the existence of an equilibrium configuration involving N firms, in a market of size S, is that $1 \leq \frac{S}{N} \leq 2$. To see this, note that the combined payoff for all the firms in the market is S (the total number of consumers). The lowest possible payoff to a firm would then be (if we consider each firm splitting the market evenly), $\frac{S}{N} - 1$. And so by the viability condition that says these payoffs in equilibrium must be positive, $\frac{S}{N} \geq 1$. At the same time, consider a set up with all firms making payoff $\frac{S}{N} - 1$ and an additional firm enters the market. The worst each firm in the market could do (those firms that have to now share the market with the entrant) is $\frac{1}{2}\frac{S}{N} - 1$. (ie, a firm that moves in right next to an established firm steals half his payoff). Thus, by the stability condition, stating that the profit after the entry is negative, $\frac{S}{N} \leq 2$. So we have two conditions:

$$\frac{S}{N} \geq 1 \Longrightarrow S \geq N.$$
$$\frac{1}{2}\frac{S}{N} \leq 1 \Longrightarrow S \leq 2N.$$

  As seen in the graph in "Technology and Market Structure" on page 43, we define upper and lower bounds for equilibrium configurations. ie, for all equilibrium configurations: S and N, $S \in (N, 2N)$.

3 Week 3: 28 Jan - 1 Feb

3.1 The setup

- The broad aim is to examine a class of models with 2 features: 1) firms are characterised by a level of capability with c denoting productivity (or some level of marginal costs so as productivity rises, c falls), and u denoting a shift parameter that can be thought of as either the quality of the good produced or consumer's willingness to pay for a product. 2) it is possible to raise capability by making fixed investments, F.

3.2 The Normal Set - Classical Market Setup

- The concept of what defines a market is rather tricky. One formal definition was that a market must be broad enough to define all products up until those that "break the chain of substitutes." This results in large markets though within them there are many "submarkets" which are goods that are close substitutes for one and other.
- Under the classical market setting, assume that we are looking at a market that is well defined and products in the market are all close substitutes. We will address the idea of submarkets later.
- Today we will examine the non-convergence theorem in terms of the classical market.
- Intuition: If goods differ in quality, then if we have one good of quality much higher than the rival qualities, the firm with the high quality good will enjoy some minimum level of market share. Firms with higher quality goods will be profitable because their demand curve is out further than the rest of the market and the firm is able to charge a quality premium above other firms.
- Even with increased competition and prices falling towards marginal cost, the high quality firm will still have his premium and will make profits.

3.2.1 A "Quality-Choice" Framework

- There is a single classical market. The notation is as follows. Firm i offers quality u_i. A configuration:
$$\tilde{u} = (u_1, u_2, \ldots, u_N).$$
- There are N active firms in the market.
- Define the top quality as $\hat{u} = max_i(u_i)$.
- Gross profit is defined as:

$$\Pi(u_i|\tilde{u}_{-i}) = S \qquad \underbrace{\pi(u_i|\tilde{u}_{-i})}_{Profit\ per\ firm\ in\ market} \qquad .$$

- Industry Sales Revenue:

$$Y(\tilde{u}) = Sy(\tilde{u}).$$

- We have some fixed costs, F, of quality improvement defined as:

$$F(u_i) = F_0 u_i^{\beta}.$$

  Where $u_i \in [1, \infty)$. Note that the basic level of quality, 1, incurrs a cost of F_0. No product can have quality below 1.

- Note also that the β coefficient measures the effectiveness of fixed outlays in raising u.

- As $\beta \longrightarrow \infty$, the fixed outlays become ineffective and here, the outcomes in the model will coincide with those of the exogenous sunk costs model. See graph in notes that shows for higher β's, the cost of fixed outlays (and therefore product improvement) is less effective. [**Graph**]

- In the light of this, we define an expenditure function that goes above and beyond the level of basic quality:

$$R(u_i) = F(u_i) - F_0.$$

  So R would include things such as $R\&D$ and advertising.

- Beyond the two assumptions introduced in defining an equilibrium configuration, add a third assumption:

$$\exists\, \eta > 0, \ni\ \forall\, \tilde{u} \neq \{\emptyset\}, y(\tilde{u}) \geq \eta > 0.$$

  Or written in a slightly different form:

$$\forall\, \tilde{u} \neq \{\emptyset\}, Sy(\tilde{u}) \geq S\eta \longrightarrow \infty \; as \; S \longrightarrow \infty.$$

  Thus, the level of industry sales revenue is bounded away from zero for all non-empty $\tilde{u}$. As the market size gets huge, industry sales increase right along with it.

- The Main Theme: We would like to show that there must be a larger sized firm in any market. To show this, we use the central notion of game theory. We'll assume that we have a very fragmented industry with many small firms and then show that this equilibrium is NOT Nash ... ie, there exists, a profitable deviation.

- So consider a configuration where you have the current firms in a market all producing some quality level, $\tilde{u}$. Then one firm decides to "jump" to a higher quality level above all the rest. We would like to define how far he will jump, K, and then what his payoff would be. So for each $K > 1$, we define an associated number $a(K)$ as follows:

$$a(K) = inf_{\tilde{u}} \frac{S\pi(K\hat{u}|\tilde{u})}{Sy(\tilde{u})}.$$

Noting that the S's cancel, we have the profit of the high quality entrant on top (delivering a quality of K times the highest current market quality to the market) and we divide this by the pre-entry industry sales revenue. Inf just means take the minimum $a(K)$ but since $\tilde{u}$ is continuous, we use inf. This allows us to get at the profits of the entrant no matter what $\tilde{u}$ looks like.

- THEOREM: NON-CONVERGENCE. Given a pair, $K > 1$, $a(K) > 0$, a necessary condition for a configuration $\tilde{u}$ to be an equilibrium configuration is that at least one firm has market share exceeding:

$$\frac{a(K)}{K^{\beta}}.$$

Or in other words,

$$C_1 \geq \frac{a(K)}{K^{\beta}}.$$

- Proof: Consider a configuration $\tilde{u}$ with maximum quality $\hat{u}$. Select a firm offering $\hat{u}$ and denote its sales revenue as $S\hat{y}$. Hence its share of industry sales revenue is:

$$\frac{S\hat{y}}{Sy(\tilde{u})} = \frac{\hat{y}}{y(\tilde{u})}.$$

Consider an entrant offering $K\hat{u}$. Its net profit is at least:

$$a(K)Sy(\tilde{u}) - F(K\hat{u}).$$

$$\Rightarrow \frac{S\pi(K\hat{u}|\tilde{u})}{Sy(\tilde{u})} Sy(\tilde{u}) - F(K\hat{u}).$$

$$\Rightarrow S\pi(K\hat{u}|\tilde{u}) - F_0(K\hat{u})^{\beta}.$$

$$\Rightarrow S\pi(K\hat{u}|\tilde{u}) - K^{\beta}F(\hat{u}).$$

Thus, from the first equation, net profits are at least

$$a(K)Sy(\tilde{u}) - K^{\beta}F(\hat{u}).$$

One of the conditions for equilibrium is stability which implies that net profits of this entrant must be non-positive. Thus,

$$a(K)Sy(\tilde{u}) - K^{\beta}F(\hat{u}) \leq 0.$$

$$a(K)Sy(\tilde{u}) \leq K^{\beta}F(\hat{u}).$$

$$F(\hat{u}) \geq \frac{a(K)}{K^{\beta}}Sy(\tilde{u}).$$

And the other condition for equilibrium, viability, implies that in the original configuration, each firm, and so in particular, the firm offering $\hat{u}$, earns sufficient sales revenue greater than its fixed outlays:

$$S\hat{y} \geq F(\hat{u}).$$

Combining these last two equations,

$$S\hat{y} > F(\hat{u}) \geq \frac{a(K)}{K^{\beta}}Sy(\tilde{u}).$$

$$S\hat{y} \geq \frac{a(K)}{K^{\beta}}Sy(\tilde{u}).$$

$$\frac{S\hat{y}}{Sy(\tilde{u})} = \frac{\hat{y}}{y(\tilde{u})} \geq \frac{a(K)}{K^{\beta}}.$$

Thus, the firm in the orginial configuration offering $\hat{u}$ must have market share at least equal to that expression.

- The Non-convergence theorem places a lower bound on C_1. It is convenient to define the minimum market share as follows:

$$\alpha = Sup_K \frac{a(K)}{K^{\beta}}.$$

We include the *Sup*, supremium, because this condition is a function of K and there are many possible values of K. Thus, the concentration index, C_1, must not fall below the highest lower bound. The Key ideas is that α is independent of S, market size. See graph in notes. [**Graph**]

3.2.2 Extensions

- With reference to chapter 3 of Technology and Market Structure, we will continue the analysis with two extensions.
- 1) So far, we have a statement that involves a and K which are extremely hard to measure. Can we translate this theorem into a statement about things that are easy to measure?
- 2) What happens when we move beyond the setting of the classical market. We will not be able to assert that there are some pairs of numbers, a and K, for the market.

4 Week 4: 4 Feb - 8 Feb

4.1 An "Ancillary" Theorem

- Following last week's "non-convergence theorem", we move to the ancillary theorem which allows the result to be formulated in terms of observables: concentration and the level of *R&D* or advertising intensity.

- To do this, we introduce a "class of models." With this in mind, we choose some fixed utility function for consumers which defines some (a, K) pair and we construct a 1-parameter family of models by allowing the β parameter in the cost function to vary.

- Motivation: The mapping of interest is the mapping of cost function, $F(\cdot)$, into the final stage gross profit, $\pi(\cdot)$. Notice we can break this down into 2 steps: $F(\cdot) \longrightarrow u \longrightarrow \pi(\cdot)$. But all results depend only on the composite mapping from $F(\cdot)$ to $\pi(\cdot)$. (ie, we can relabel u, replacing it by any monotonic function of u.)

- Note the non-convergence theorem from last week

$$C_1 \geq \alpha = Sup_K \frac{a(K)}{K^\beta}.$$

  Taking the $(a(K), K)$ pair as given, we want to investigate changes in β. So we need to formally link β to the level of *R&D* intensity.

- Before proceeding to develop the ancillary theorem, we need to address a technical point. This relates to the "Bertrand Limit." Recall that we aim to analyze the profitability of entry, but if the underlying model is the Bertrand model, then there will be a monopoly solution at equilibrium because entry causes price to fall to marginal cost. Thus for any positive level of setup costs, entry will be unprofitable. Thus no entry occurs. So, if the model collapses to the Bertrand Limit, then the lower bound to concentration is $C_1 = 1$. (ie, there can only exist a monopoly). In what follows we can exclude this case in investigating the question: Is there a lower bound to C_1? (Note that it's not that we want to avoid the Bertrand model of price competition, but rather just the Bertrand Limit.)

- Now, a figure. See notes for this key figure. **[G-4.1]** We would like to illustrate the profitability of an entrant as a function of the entrant's relative quality. We consider an equilibrium in the final stage subgame. Choose any firm already in the market as the reference firm. Call the reference firm, firm i, and firm i has profit π_i. Thus we plot the relative quality of the entrant, K, in our model on the horizontal axis plotted from $K = 0$ to 1. On the vertical axis, we plot the relative profitability of the entrant also over the $(0, 1)$ range. Labeling the entrant as firm $N + 1$, his relative profitability is:

$$\frac{\pi_{N+1}(K)}{\pi_i}.$$

In the bertrand case, even when the relative quality of the entrant is 1, the highest possible, the relative profitability is 0 because $\pi_{N+1}(K = 1) = 0$. However, without Bertrand price competition, if $K = 1$, then the relative profitability will be somewhere on the right edge of the box strictly positive but less than 1 because in general, entry must decrease profitability. As the relative quality of the entrant falls from 1 towards 0, the relative profitability falls until at some point, for K small enough, the relative profitability becomes zero or negative.

- Hence we move to Assumption 4: There exists some triple (x, γ, d) with $x > 0$, $\gamma < 1$, and $0 < d < 1$, with the following property: Suppose any firm, i, attains quality u_i and earns final stage profit that exceeds some fraction, x, of its sales revenue. Then an entrant attaining a quality level equal to $max(1, \gamma u_i)$ earns a final stage profit of at least $d\pi$. [Note that u_i is defined over $[1, \infty)$ and $d = \pi_{N+1}/\pi_i$ and thus $d\pi_i = \pi_{N+1}$.]

- Discussion of Assumption 4: So consider an industry with a lot of $R\&D$ being invested by all firms, but where the $R\&D$ is really not very effective in raising quality by all that much. Thus, it makes sense for a firm to enter the industry and invest a lot less for only a slight loss in quality. They would become profitable at a lower cost then their rivals. In other words (From tech and market structure): "If the price-cost margin earned on some currently available product is high, a firm offering a slightly lower quality can earn a postentry profit that exceeds some fraction, d, of the rival product's preentry profit." Thus, the "x" variable gives us the high cost margins, the $max(1, \gamma u_i)$ reflects the fact that the quality of products has to be at least 1, so an entrant who undercuts the quality of firm i (by a factor γ) cannot provide a quality lower than 1. The point (γ, d) lies inside the curve on the graph. Note that as the price competition becomes more fierce, the curve itself shifts down and to the right so making the possible combinations to find a point (γ, d) more limited. ie, an entrant can't force his way into the market because he has to have a high relative quality and low or zero profits post-entry. Thus, it's only in the situations where this curve is drawn as it is in the notes that an entrant can come in and undercut the quality.

- THEOREM: Ancillary theorem. For any threshold level of $R\&D$ to sales ratio exceeding $max(x, 1 - d)$ where x and d are defined as above, there exists an associated value, β^* such that for $\beta > \beta^*$, no firm can have an $R\&D$ to sales ratio exceeding this value, x, in any equilibrium configuration.

- Here, we are basically taking the idea introduced in assumption 4 and making it applicable by putting it in terms of the β paramater of the cost function. So if $R\&D$ investments are ineffective, then we can also find the associated cost parameter which measures the effectiveness of fixed outlays. Since as β rises, the effectiveness of fixed outlays falls, we say that for any β greater than some β^*, there will exist an opportunity for a firm to come in and undercut on quality. If this opportunity exists, the configuration must not be an equilibrium.

- See graph in notes which superimposes the cost functions on the graph from earlier. **[G-4.2]** Note for our (x, γ, d) combination, we also have a β^* value which corresponds to that point. For values of $\beta > \beta^*$, entry is possible by some lower quality entrant.

- Recipe for an empirical test: Take a number x as a threshold level of $R\&D$ to sales. The assumptions allows us to find an associated point (γ, d) and an associated point β^*. So if we split the industries into two groups, those with $R\&D$ to sales ratios above x, and those with $R\&D$ to sales ratios below x, all the industries with $\beta > \beta^*$ will lie in the low $R\&D$ intensity group. (ie, for the industry to be in equilibrium, then having a high β should go along with low $R\&D$ intensity.) And conversly, all industries living in the high $R\&D$ group will have $\beta < \beta^*$ and so we get the result:

$$C_1 \geq \alpha \geq Sup_K \frac{a(K)}{K^{\beta^*}}.$$

4.1.1 A Linear Demand Model

- Consider a market with the features of both vertical (quality) and horizontal (substitutability) product differentiation. We will need a 2-parameter family. Consider a linear demand model, the cournot version. Variety will now be the choice variable of the firm. Assume consumers maximize the following utility function:

$$U(x_1, ..., x_N) = \sum_{k=1}^{N} (x_k - x_k^2) - 2\sigma \sum_k \sum_{l \leq k} x_k x_l + M.$$

  Where σ is the substitutability parameter with $\sigma = 1$ impliying that all goods are equivalent (perfect substitutability) so utility equals the sum of the quantities of all the goods. M is the money income spent on other goods (the Hicksian composite commodity). Note to get a linear demand function, we need a quadratic utility function for obvious reasons.

- Consumer's Problem: Maximize U subject to the budget constraint: $M = Y - \sum_k p_k x_k$.

- At equilibrium, the consumers spend money on all available varieties and a lower price for any variety will increase the consumption of that variety (nicely behaved equilibrium).

- The resulting linear demand function is as follows:

$$p_k = 1 - 2x_k - 2\sigma \sum_{l \neq k} x_l.$$

  Note the demand curve shifts up as σ falls. Thus, as a certain good becomes more and more non-substitutable, the demand for it rises. This makes intuitive sense as consumers are more willing to pay higher prices for a good with few substitutes.

- NEXT WEEK: Using the example, we examine first the exogenous sunk cost case. Here a firm can introduce any number, n_i, of varieties at cost of ϵ per variety.

- We will augment the utility function by introducing a "quality" parameter, u_k, for good k to reach the general case of endogenous sunk costs.

5 Week 5: 11 Feb - 15 Feb

5.1 More on the Linear Demand Model

- Recall the model in which preferences were modeled such that the resulting demand equations were linear. From this analysis, we can derive viability and stability conditions as shown in the graph in the notes [**G-5.1**]. We have the usual inverse relationship between the number of varieties of goods on the horizontal axis and C_1 on the vertical axis. The stability condition puts a lower bound on the number of products for each level of concentration and the viability condition puts an upper bound. The point, x, corresponds to the minimum level of concentration possible in the market. Notice as $S \to \infty$, both the stability and viability conditions shift to the right and drive x down so the concentration index, C_1, converges to its lower bound.

5.2 Beyond the Classical Model

- Motivation: Traditional literature tries to "explain" C_k by regressing C_k on some candidate variables. One such regression would be:

$$C_k = a + b\left[\frac{\epsilon}{Y}\right] + \frac{A}{Y} + \frac{R\&D}{Y}.$$

  The first term, ϵ/Y, measures the degree of scale economies in an industry. ϵ is the cost of a single plant of minimal efficient scale and Y is total industry sales. There is a clear positive relation between this term and C_k. The second term, A/Y measures the advertising to sales ratio. This usually shows up as a weak positive relationship, however we normally consider it endogenous to the model so it's probably misspecified. The final term adds nothing to the analysis. It's never significant.

- Another possible method would be to simply compute the correlation between C_k and the $R\&D$ to sales ratio. The relevant reference is in "Handbook of IO" (1991): an article by Cohen and Levin. The results of this type of analysis, as expected, are disappointing. The resulting relationships are shown in the graph in the notes [**G-5.1**]. When you also include dummies for type of industry, the model lacks all explanatory power. The point of this is that there are other important control variables that needed to be accounted for. This is what we seek to do today.

- Motivation (part 2): Interpreting α. Consider an ice cream seller in the hotelling model who improves the quality of his product. What will happen to the hotelling umbrellas? Consider utility functions of the form:

$$U = Constant - p + u - t \cdot d.$$

  Note we are now including a quality component, u. Do to the form of the utility function, an increase in u is equivalent to a decrease in p. Thus, we could alter the umbrellas by shifting down the higher quality umbrella and thus increasing that firm's market share along the beach. This effect, the resulting increase in sales resulting from a high quality is embodied in the β paramter in the cost function. It measures the efficiency of investment.

- However, there is also another component that needs attention. Consider the slopes of the umbrellas. As shown in the notes [**G-5.2**], goods that are poor substitutes will have steeper umbrella arms. If the the umbrellas are relatively steep, then the resulting increase in market share from the quality improvement will be lessened. The degree of substitutability in the industry will be parameterized with the value σ.

- Returning to the linear demand analysis, we will extend the model by introducing quality and then we get a non-convergence result as before. The new idea that will emerge is that the lower bound to the concentration index $\underline{C}_1$ will depend on two parameters: β and σ. The KEY result: As $\sigma \to 0$, $\underline{C}_1 \to 0$. (?)

5.2.1 Two examples

- Consider the market for airplanes. Although airplanes themselves are highly differentiated, buyers only care about one thing: the unit cost of carrying a passenger. Thus, in this industry σ is fairly high (as is α). There are large returns from investment and innovation.

- The opposite would be true in the market for Flow Meters. These things measure the rate of flow of chemicals through a plant for example. There are many types of flow meters and usually each plant will require a very specific type of meter. Here σ would be fairly low (as would α) and for a firm to enter the industry, they are best just to develop a new type of meter and sell it to a particular segment of the market.

- Another analogy to the low α industries is a lottery. Explained on page 71 of Technology and Market structure, a high spending player in a lottery can always have his expected profits erroded by just increasing the number of low spending players. "The example illustrates by analogy what is going on in a low-alpha industry: The high-spending entrant CANNOT achieve a profit exceeding some fixed poportion of current industry sales independently of the number of low-spending rivals."

- Thus, in the airplane industry, we can get an equilibrium with high $R\&D$ to sales ratios and high concentration. In the flow meter market, we can also get high $R\&D$ to sales ratios but along with low concentration. It all depends on the dynamics of the market.

5.2.2 The General Model

- We now extend the general analysis to a setting where the market contains various submarkets. See graph in notes [**G-5.3**].

- The outer bound defines a break in the chain of substitutes. However as a subset of this larger set, we have a series of submarkets where the goods in each submarket are "close" substitutes. Whether there is substitutability between submarkets, we leave this point open for the moment.

- Suppose a x firm spends a lot on $R\&D$, can he steal customers from a y firm? The answer to this question is embodied in σ. If $\sigma = 0$, then the goods are not-substitutable between submarkets and one firm's expenditure on $R\&D$ will not have an effect on a firm in a different submarket. (though this seems in contradiction with the outer boundry of the classical market: ie, this would seem to imply a break in the chain of substitutes.)

- We introduce a 2-parameter family of models where $\alpha = \alpha(\beta, \sigma)$. As before, β parameterizes the cost function, $F(u) = F_0 u^{\beta}$. But here's the new bit: We allow firms to invest in $R\&D$ along several alternative trajectories and to sell in a number of associated submarkets.

- Firm i's capability is defined by an M-tuple,

$$\tilde{u}_i = (u_{i1}, u_{i2}, \ldots, u_{iM}).$$

  Where u_{ij} is firm i's quality/capability in submarket j. Thus there are a total of M submarkets.

- The assumptions that follow serve to define what is meant by a technical trajectory and its associated market.

- Assumption 5: Defining the idea of a submarket: There is a pair (a_0, K_0) with $a_0 > 0$ and $K_0 > 1$, such that in any configuration, $\tilde{u}$ with maximum quality, $\hat{u}$ on trajectory m, an entrant offering quality, $K_0 \hat{u}$ along trajectory m will achieve a final stage profit of at least $a_0 S y_m(u)$. The idea here is that the high quality product captures market share ONLY in its OWN submarket.

- Assumption 6: Introduces the substitution parameter, σ.

  - i.) For any $\sigma \geq 0$,

$$y_m(u^{(m)}) \geq y_m(\tilde{u}).$$

    First note that small y_m is per capita sales of the industry in submarket m. $\tilde{u} = (\tilde{u}_1, \tilde{u}_2, \ldots, \tilde{u}_N)$. Thus $\tilde{u}$ is the capability matrix, (MxN) of all N firms along each of the M trajectories. So what is $u^{(m)}$? Define $u^{(m)}$ as that in which all firms capabilities on trajectory m are the same as in $\tilde{u}$ but all capabilities on other trajectories (besides m) are set to 0. So for firm i,

$$u^{(m)} = (u_1, u_2, \ldots, u_m, \ldots, u_M) = (0, \ldots, 0, u_m, 0, \ldots, 0).$$

    See text for clarification, but basically this means that if goods are at all substitutable between submarkets, then the sales in submarket m from investing only along the m trajectory cannot be smaller than the sales in submarket m from a configuration where firm i invests along other trajectories. In other words, (page 80), "removing products in other categories does not diminish the sale of products in category m." If anything, say σ is strictly positive, if you are removing substitutes from the market, the demand for the remaining good should rise. Hence we have the following:

$$\text{When } \sigma = 0, \text{ then } y_m(u^{(m)}) = y_m(\tilde{u}).$$

Thus, if we have independent submarkets, then removing the other goods has no effect on sales in category m.

– ii.) As $\sigma \to 0$ the ratio,

$$\frac{y_m(\tilde{u})}{y_m(u^{(m)})} \to 1 \text{ Uniformly in } \tilde{u}.$$

This is just a technical point so that we get continuity. It is not only at $\sigma = 0$, that we have independent markets, but as we approach independent markets (ie, goods become increasingly non-substitutable), the sales in market m becomes independent of what's happening in the other submarkets.

- Assumption 7: For any $\sigma > 0$ (linked submarkets), $\exists$ a quality ratio, $\gamma_0 \in (0, 1]$, such that a product of quality $\gamma\tilde{u}$, with $\gamma \leq \gamma_0$, cannot command positive sales revenue if a rival firm offers a product of quality $\tilde{u}$ on any trajectory. [NOTE: Notationally, $\tilde{u}$ is not a vector but rather a constant in this case (done to stay consistent with the text).] See graph in notes **[G-5.4]** which sets this up. If submarkets are linked, a high quality product might put a lower quality firm in another submarket, out of business. The key point is that if $\sigma = 0$, (perfect non-substitutibility), then $\gamma_0 \to 0$, and this assumption (may) fail. That is, if $\sigma = 0$, a low quality product may survive no matter how good or how high quality are the products in the other submarkets.

- Next week we will move from a model parameterized by (β, σ) to a model of observables. We will examine the $R\&D$ to sales ratio and a homogeneity index. See chapter 3 (4?) of Technology for background material.

6 Week 6: 18 Feb - 22 Feb

6.1 More on the 2 Parameter Model

- From last weeks assumptions on our 2 parameter class of models we need to define a new variable:

$$h = Max_m \ \frac{y_m(\tilde{u})}{y(\tilde{u})}.$$

  Where h is the share of total industry sales accounted for by the largest submarket.

- The motivation behind this is that submarket structure emerges as an endogenous outcome given the values of β and σ.

- When h is small, the market is very fragmented with many smaller submarkets. When h is big, we have one firm or product group dominating the market and all others have a relatively small share of the market.

- Theorem 3: (Extension of Non-Convergence). In an equilibrium configuration, the one firm sales concentration ratio satisifies:

$$C_1 \geq \frac{a_0}{K^\beta} \cdot h.$$

  Thus h, C_1 and α are all positively related.

- Ancillary Theorem. As before this links the $R\&D$ to sales ratio to the β coefficient. The proof is tedious and thus omitted.

- Recall the empirical prediction for the single "Classical Market." See graph in notes that shows for industries with low $R\&D$ to Sales ratios, the C_1 figure can converge to 0 as S goes to infinity. When $R\&D$ to sales ratios are high, there exists a lower bound to C_1, namely, $\underline{C}_1$. **[G-6.1]**

- Predictions for the general case. Again see graphs in notes. If $R\&D$ to sales ratios are low, we can get any degree of concentration for any level of product homogeneity, h. If $R\&D$ to sales is high, then if h is low, C_1 can be anything, but as h increases, the C_1 level rises. So, in this case, when there are large returns to research and investment, then if one product is taking a large part of the market, it is necessary that one firm will become the dominate firm in the market and capture a large market share.

6.2 Empirical Testing

- There are two types of tests we can use to test the predictions we stated in the last section. Statistical tests and Case Histories.

- Consider a classical submarket so there exists demand side linkages. We look at industries that have segments that are advertising intensive because we can almost determine the exact scope of one firm's advertising expenditure. Thus the relavent markets are well defined. We allow markets that are defined narrowly enough to allow a brand image to span the market.

- A study looked at 20 industries all in the food and drink sector and compared those with high advertising to sales ratios and those with low ads to sales ratios.

- See graph in notes [**G-6.2**] which shows that for markets with low advertising to sales ratios, the concentration ratio converges to zero. In advertising intensive industries, the concentration index is bounded significantly above zero.

- In terms of a test for the general case, we examine high and low $R\&D$ to sales ratio industries in the US in 1977. We find that concentration increases with h in high α industries and when α is low, any concentration is seen in equilibrium for all values of h. [**G-6.3**]

- Beyond the statistical tests, we'd like to test the fundamental principals of the theory to see if these statistical relationships are actually a result of the theory. To do this we use Case Histories which are a rather controversial part of economics because not many put much faith in them. However, they should be viewed as complementary to the statistical tests.

- Key Theoretical Principals: If you have a set of outcomes and a subset of Nash Equilibriums, if you find a configuration, x, such that x is outside the set of NE, there must exist a profitable deviation. It is intrinsic to any game theoretic model that it must specify the qualitative nature of this profitable deviation. So the idea is to find the x and see if firms in that situation are acting on the profitable deviation. [**G-6.4**]

- What the theory does not specify is the path along which the industry adjusts to the new equilibrium. This is part of dynamic games and results are in general NOT ROBUST. This is because the problem of unobservables such as the beliefs of the agents.

- The best case studies are those that are Natural Experiments. We have the setup where x is initially in the NE subset of outcomes, but then something happens externally so that the NE subset shifts. x now lies outside of the NE subset. We look to see what happens to x as he moves back into NE.

- A good example of a natural case is the salt industry in late 1800's. Suddenly through the evolution of the transportation system (railways), markets once geographically seperated were suddenly competing with one and other. This is basically the flattening of Hotelling's Umbrellas. What happened was a series of mergers and acquisitions between competing firms which raised C_1. The key variable in all of this is the toughness of price competition. So as firms started competing with one and other, the toughness of price competition rose, thus we see higher concentration, meaning that C_1 has risen.

7 Week 7: 25 Feb - 1 Mar

7.1 Natural Experiments

- The following example will involve the Escalation Mechanism. Consider the Frozen Food industry. One institutional feature is that output is sold through two channels. The Retail sector (directly to consumers) and the non-retail sector (hotels, catering, etc.) The model predits a Dual structure with a small number of high advertising firms and many small firms that do not advertise. Which group will be more profitable is the crucial feature.

- The crucial stage in the game is where all the big firms are advertising a lot and making large profits. Very small firms are advertising very little and making slightly less profits. The middle segment involves firms that try to invest in advertising but since their market share is too small, their profits are the lowest of the 3 groups. Thus in this crucial stage, we see a migration up and down in terms of firm sizes. Since some firms are moving towards the "big" end, we see an increase in the concentration index, eg, an escalation mechanism. See graph in notes. **[G-7.1]**

- It is interesting that in the US and UK, the first economies with this industry, this middle stage existed and we saw a migration up and down. By the time other markets opened up in other economies, they had seen what had happened in the US and UK and the structure of the market became immediately concentrated.

7.2 Aside on Scope Economies

- So far, we interpreted σ as capturing linkages on the demand side via substitution effects. We can get important linkages on the supply side via scope economies in $R\&D$. A firm that is spending to make one of its products better may, at the same time, make other products that it produces also better from this investment.

- Modeling: If firm i spends F_m on trajectory m, this raises u_m and also has positive spillover effects in raising u_j for $j \neq m$.

- Note the assumptions on σ above carry over to this setting.

- The key idea: Can a low u product on trajectory j survive in the presence of a high u product on trajectory m? If σ is large, then NO because for the high spending firm in submarket m would also have high quality in submarket j.

- The implication for the basic statistical prediction is shown in the graph in the notes. When scope economies do not exist: σ very low, then the concentration index increases in h. When scope economies exist: If h is low, there is a lower bound to the concentration index. As h rises, C_1 does as well. **[G-7.2]**

7.3 Natural Experiments (Continued)

- Beyond the classical market, recall we write α as a function of β and σ.
- First consider a natural experiment on β. Recall that β measures the "effectiveness of raising consumer's willingness to pay for a product." If β is high, the $F(u)$ function is steeply sloped, and $R\&D$ spending is ineffective. If β is low, the $F(u)$ function is shallow and $R\&D$ is effective. See graph in notes. [**G-7.3**]
- Consider the market for photographic film. In the pre-1960 era, all film was black and white. After 1960, color film began to take over the market. Initially the quality of the color pictures was poor. More importantly, with black and white film, there were really no returns to increased $R\&D$ as the technology was very standard and universal. With the introduction of color film, β fell so $R\&D$ becomes more effective because there were improvments to be made in the color film.
- So firms were faced with two possible strategies: spend a lot on $R\&D$ to capture the market, or get out because black and white film was going to become obsolete.
- See Sunk Costs chapter 7 for details, but what happened was that there were two developing techniques, oil and water based. Kodak and Fuji went with the higher quality oil based and took over the market. Other firms couldn't compete. Illford, a UK company, tried to go with water based and even tried to avoid the developing problems by raising the price of their film and then developing it for the customer. The UK monopolies commission broke up Illford due to the allegation of vertical restraints in the film production - film development industry. Illford now only does black and white film.
- So a fall in the level of β led to a higher level of concentration.
- Now consider the σ parameter. Increasing the linkages between submarkets raises σ and therefore α. Thus, markets become more concentrated.
- Consider the Telecom switches market. Initally the industries were highly protected (σ low) so there was little competition between countries. Then in the 1970's the US decided to deregulate and the world all followed suit. The increased linkages caused concentration to rise substantially as it came at the same time that companies were switching to digital switches which required a large $R\&D$ expenditure.
- $\beta \searrow \Longrightarrow \alpha \nearrow$.
- $\sigma \nearrow \Longrightarrow \alpha \nearrow$.

7.4 An Extension - Learning Model

- So far, we considered $F(u)$ as representing $R\&D$ or advertising. We now consider two further applications. Network Externalities and Learning by Doing.

- Consider a 2 stage game where in period 1, firms undertake some sort of costly investment which effects their profitability in stage 2.
- In a learning by doing model, higher output in period t_1, implies lower unit cost in period t_2.
- This model develops a clear analogy between the nonconvergence results discussed earlier and a similar result that emerges under learning by doing.
- Suppose N firms offer a homogeneous product to a population of consumers whose inverse demand function takes the form:

$$p = \frac{S}{\sum_j x_j}.$$

With S denoting the total expenditure per period on this good.

- All firms have constant marginal cost of 1 in period 1 and then in period 2, firm i's marginal cost is constant at some level $c_i \leq 1$ that depends upon the output x_i the firm sets in period 1. Thus,

$$c_i = min(1, x_i^{-\theta}), \quad \theta \geq 0.$$

The parameter, θ, measures the strength of the learning effect. When $\theta = 0$, the firm's marginal cost is constant at 1 over both periods. For all $x_i > 1$, the elasticity of marginal cost with respect to first period output is:

$$\frac{x_i}{c_i}\frac{dc_i}{dx_i} = \frac{x_i}{x_i^{-\theta}}(-\theta x_i^{\theta-1}) = -\theta.$$

Hence we see how θ reflects the marginal cost savings that firm will have in period 2 by providing more output in period 1.

- Firm i's profits:

$$S\pi_i = (p - c_i)x_i.$$

- FOC of profit function with respect to x_i:

$$S\frac{d\pi}{dx_i} = p + x_i\frac{dp}{dx_i} - c_i = 0.$$

$$= p - x_i\frac{S}{(\sum_j x_j)^2} - c_i = 0.$$

- Solving for x_i:

$$p - x_i\frac{S}{(\sum_j x_j)^2} - c_i = 0.$$

$$x_i \frac{S}{(\sum_j x_j)^2} = p - c_i.$$

$$x_i = (p - c_i)\frac{(\sum_j x_j)^2}{S}.$$

- Summing over all N firms,

$$\sum_j x_j = (Np - \sum_j c_j)\frac{(\sum_j x_j)^2}{S}.$$

Substituting out p from above,

$$\sum_j x_j = (N\frac{S}{\sum_j x_j} - \sum_j c_j)\frac{(\sum_j x_j)^2}{S}.$$

$$\sum_j x_j = N\sum_j x_j - (\sum_j c_j)\frac{(\sum_j x_j)^2}{S}.$$

Solving for $\sum_j x_j$:

$$1 = N - (\sum_j c_j)\frac{\sum_j x_j}{S}.$$

$$(\sum_j c_j)\frac{\sum_j x_j}{S} = N - 1.$$

$$\sum_j x_j = \frac{S(N-1)}{\sum_j c_j}.$$

- Prices:

$$p = \frac{S}{\sum_j x_j} = \frac{S}{\frac{S(N-1)}{\sum_j c_j}} = \frac{\sum_j c_j}{N-1}.$$

- Now we can substitute $\sum_j x_j$ into firm i's output decision:

$$x_i = (p - c_i)\frac{(\sum_j x_j)^2}{S}.$$

$$x_i = (p - c_i)\frac{\left(\frac{S(N-1)}{\sum_j c_j}\right)^2}{S} = (p - c_i)\left[\frac{N-1}{\sum_j c_j}\right]^2 S.$$

- And finally back to the profit function of firm i:

$$S\pi_i = (p - c_i)x_i.$$

$$S\pi_i = (p - c_i)(p - c_i)\left[\frac{N-1}{\sum_j c_j}\right]^2 S = (p - c_i)^2\left[\frac{N-1}{\sum_j c_j}\right]^2 S.$$

Note the price equation above: $p = \dfrac{\sum_j c_j}{N-1}$. Subtracting c_i from both sides:

$$p - c_i = \frac{\sum_j c_j}{N-1} - c_i.$$

Substituting $p - c_i$ into the profit function above:

$$S\pi_i = (p - c_i)^2\left[\frac{N-1}{\sum_j c_j}\right]^2 S.$$

$$S\pi_i = \left[\frac{\sum_j c_j}{N-1} - c_i\right]^2\left[\frac{N-1}{\sum_j c_j}\right]^2 S.$$

Simplifying,

$$S\pi_i = \left[\left[\frac{\sum_j c_j}{N-1} - c_i\right]\left[\frac{N-1}{\sum_j c_j}\right]\right]^2 S.$$

$$S\pi_i = \left[1 - c_i\left[\frac{N-1}{\sum_j c_j}\right]\right]^2 S.$$

$$S\pi_i = \left[1 - (N-1)\frac{c_i}{\sum_j c_j}\right]^2 S.$$

There it is.

- Now if all firms have the same marginal cost in the second period (c_i), the profit per firm reduces to:

$$S\pi_i = \left[1 - (N-1)\frac{c_i}{Nc_i}\right]^2 S.$$

$$S\pi_i = \left[1 - \frac{N-1}{N}\right]^2 S.$$

$$S\pi_i = \left[\frac{1}{N}\right]^2 S = \frac{S}{N^2}.$$

- Now we would like to see how profits obtained in period 2 relate to each firm's marginal cost. To do this, take the derivative of the profit function and use it to find the elasticity. Thus,

$$\frac{c_i}{\pi}\frac{d\pi}{dc_i} = \left[\frac{c_i}{\left[1-(N-1)\frac{c_i}{\sum_j c_j}\right]^2}\right] 2\left[1-\frac{(N-1)c_i}{\sum_j c_j}\right] \cdot \frac{\sum_j c_j(N-1)-(N-1)c_i}{(\sum_j c_j)^2}.$$

Now let $c_i = c_j = c$. Thus,

$$\frac{c_i}{\pi}\frac{d\pi}{dc_i} = \left[\frac{c}{\frac{1}{N^2}}\right] 2\left[\frac{1}{N}\right] \cdot \frac{Nc(N-1)-(N-1)c}{(Nc)^2}.$$

$$= \left[cN^2\right] \cdot \left[\frac{2}{N}\right] \cdot \frac{Nc(N-1)-(N-1)c}{(Nc)^2}.$$

$$= \frac{2}{N}(N(N-1)-(N-1)) = \frac{2N^2-2N-2N+2}{N} = 2N-4+\frac{2}{N} = -2\frac{(N-1)^2}{N}.$$

- To solve the problem, we need to consider what x_i that firms will choose in period 1 that will maximize the sum of their profits in periods 1 and 2. Thus total profits equal:

$$\Pi_i = (p-1)x_i + S\pi.$$

Where the first term is period 1 profits (unit marginal cost) and the second term is the second period profits defined above. Substituting in,

$$\Pi_i = (\frac{S}{\sum_j x_j} - 1)x_i + \left[1-\frac{c_i(N-1)}{\sum_j c_j}\right]^2 S.$$

- FOC (with respect to x_i):

$$\frac{d\Pi_i}{dx_i} = \left[\frac{S}{\sum_j x_j} - \frac{Sx_i}{(\sum_j x_j)^2} - 1\right] + S\frac{d}{dc_i}\pi(c_i|(c_{-i})) \cdot \frac{dc_i}{dx_i} = 0.$$

Note we can rewrite the final term as $\frac{d\pi}{dc_i}\frac{dc_i}{dx_i} = (\frac{c_i}{\pi}\frac{d\pi}{dc_i})(\frac{\pi}{c_i}\frac{dc_i}{dx_i})$. Thus this term solves to:

$$\frac{d\pi}{dc_i}\frac{dc_i}{dx_i} = -2\frac{(N-1)^2}{N} \cdot \frac{\pi}{x_i^{-\theta}}(-\theta x_i^{-\theta-1}).$$

$$= 2\theta\Big(N-2+\frac{1}{N}\Big)\frac{\pi}{x_i}.$$

So substituting this into the FOC of Π_i yields:

$$\frac{d\Pi_i}{dx_i} = \left[\frac{S}{\sum_j x_j} - \frac{Sx_i}{(\sum_j x_j)^2} - 1\right] + S\frac{d}{dc_i}\pi(c_i|(c_{-i})) \cdot \frac{dc_i}{dx_i} = 0.$$

$$\left[\frac{S}{\sum_j x_j} - \frac{Sx_i}{(\sum_j x_j)^2} - 1\right] + 2S\theta\Big(N - 2 + \frac{1}{N}\Big)\frac{\pi}{x_i} = 0.$$

$$\frac{S}{\sum_j x_j}\left(1 - \frac{x_i}{\sum_j x_j}\right) - 1 + 2\theta\Big(N - 2 + \frac{1}{N}\Big)\frac{S\pi}{x_i} = 0.$$

Now in the symmetric equilibrium, $S\pi = S/N^2$ and all firms produce the same amount so $x_i = x$. Thus,

$$\frac{S}{Nx}\left(1 - \frac{x}{Nx}\right) - 1 + 2\theta\Big(N - 2 + \frac{1}{N}\Big)\frac{S/N^2}{x} = 0.$$

$$x = \frac{S}{N}\left(1 - \frac{1}{N}\right) + 2\theta\Big(N - 2 + \frac{1}{N}\Big)\frac{S}{N^2}.$$

- Now we can find the firm's period 1 profits by substituting x into $x(p-1)$. Thus,

$$(p-1)x = (\frac{S}{\sum_j x_j} - 1)x = \frac{Sx}{Nx} - x = \frac{S}{N} - x.$$

And substituting in x,

$$\frac{S}{N} - \frac{S}{N}\left(1 - \frac{1}{N}\right) - 2\theta\Big(N - 2 + \frac{1}{N}\Big)\frac{S}{N^2}.$$

$$\frac{S}{N^2} - 2\theta\Big(N - 2 + \frac{1}{N}\Big)\frac{S}{N^2}.$$

- Thus since the firm's second period profits equal $\frac{S}{N^2}$, the firm's total profits are:

$$\Pi = \frac{S}{N^2} - 2\theta\Big(N - 2 + \frac{1}{N}\Big)\frac{S}{N^2} + \frac{S}{N^2}.$$

$$\Pi = \frac{2S}{N^2}\left(1 - \theta\Big(N - 2 + \frac{1}{N}\Big)\right).$$

- We now consider the firm's entry decision. Firm i's payoff equals $\Pi - \epsilon$ where ϵ denotes a fixed and sunk cost of entry which we will set equal to 1. The equilibrium number of firms is given by equating the profit level to unity. Whence,

$$\Pi = \frac{2S}{N^2}\left(1 - \theta\Big(N - 2 + \frac{1}{N}\Big)\right) = 1.$$

$$2\left(1 - \theta\Big(N - 2 + \frac{1}{N}\Big)\right) = \frac{N^2}{S}.$$

- Finally, this equation is plotted in the notes. With N on the horizontal axis, we see there is a bound to the number of firms in the industry as S rises.

- Note that when $\theta = 0$ (no Learning by Doing), then firms earn $\frac{S}{N^2}$ in each period. So doubling it and setting it equal to the unitary setup cost yields:

$$\frac{2S}{N^2} = 1 \Longrightarrow C_1 = \frac{1}{N} = \frac{1}{\sqrt{2S}}.$$

  So $C_1 \longrightarrow 0$ as $S \longrightarrow \infty$.

- However, for $\theta > 0$, we obtain a nonconvergence result. The left side of the equation above is equal to zero when,

$$N - 2 + \frac{1}{N} = \frac{1}{\theta}.$$

  So it is clear (from the diagram?) that:

$$\underline{C}_1 \geq \frac{1}{\tilde{N}(\theta)}.$$

  Hence Non-Convergence.

- Note this model is directly analogous to the quality competition (product innovation) model of chapter 3 in Sunk Costs and Market Structure except that the quality index u is here replaced by the cost index $(1/c)$.

7.5 An Extension - Network Externalities

- In a network externality model, higher output in period t_1, implies an increase in willingness to pay (u) in period t_2.

- Consider the network externality example (Chapters 14 and 15 of Technology and Market structure). Assume the following utility function from the "Basic Cournot model with Quality."

$$u = (ux)^{\delta} z^{1-\delta}.$$

Where x is the quantity and u is the quality of the quality good, and z is the quantity of the outside good.

- Given any vector of qualities and associated prices, the consumer chooses a good that maximizes the quality / price ratio, $\frac{u_i}{p}$. Denote total expenditure on the quality good as S. It follows that all goods that command positive sales at equilibrium, must have prices proportional to their qualities. Therefore, there exists a constant, λ, such that,

$$p_i = \lambda u_i.$$

  Note that λ is a constant for all goods offered. There are clearly different quality / price combinations that will come to market, but the ratio between the two characteristics must be the same for all goods. Otherwise consumers would switch to either the cheaper or higher quality good (relative to the other characteristic).

- Expenditure, with substitution:

$$\sum p_i x_i = \sum \lambda u_i x_i = S.$$

  Thus,

$$\lambda = \frac{S}{\sum u_i x_i}.$$

- Profits for firm i (producing x_i, a good of quality u_i) is as follows:

$$S\pi_i = p_i x_i - c x_i.$$

  Where c is a constant marginal cost. Substituting,

$$S\pi_i = \lambda u_i x_i - c x_i.$$

- FOC (x_i):

$$S\frac{d\pi_i}{dx_i} = \lambda u_i + u_i x_i \frac{d\lambda}{dx_i} - c = 0.$$

  So to complete this FOC, we need the derivative of λ with respect to x_i. Recall,

$$\lambda = \frac{S}{\sum u_i x_i}.$$

  Thus,

$$\frac{d\lambda}{dx_i} = -\frac{S}{\left(\sum u_i x_i\right)^2} \frac{d}{dx_i}\left(\sum u_i x_i\right) = -\frac{S u_i}{\left(\sum u_i x_i\right)^2} = -\frac{u_i}{S}\lambda^2.$$

Substituting this into the FOC of the profit function,

$$S\frac{d\pi_i}{dx_i} = \lambda u_i - u_i x_i \frac{u_i}{S}\lambda^2 - c = 0.$$

$$\lambda u_i - c = u_i x_i \frac{u_i}{S}\lambda^2.$$

$$u_i x_i \frac{u_i}{S} = \frac{u_i}{\lambda} - \frac{c}{\lambda^2}.$$

$$u_i x_i = \frac{S}{\lambda} - \frac{cS}{u_i \lambda^2}.$$

Summing over all products,

$$\sum_j u_j x_j = \frac{NS}{\lambda} - \frac{cS}{\lambda^2}\sum_j \frac{1}{u_j}.$$

Note that $\sum_j u_j x_j = \frac{S}{\lambda}$ from above. Substituting,

$$\frac{S}{\lambda} = \frac{NS}{\lambda} - \frac{cS}{\lambda^2}\sum_j \frac{1}{u_j}.$$

Solving for λ,

$$S = NS - \frac{cS}{\lambda}\sum_j \frac{1}{u_j}.$$

$$\frac{cS}{\lambda}\sum_j \frac{1}{u_j} = NS - S.$$

$$\frac{c}{\lambda}\sum_j \frac{1}{u_j} = N - 1.$$

$$\frac{c}{\lambda} = \frac{N-1}{\sum_j 1/u_j}.$$

$$\frac{1}{\lambda} = \frac{N-1}{c\sum_j 1/u_j}.$$

$$\lambda = \frac{c}{N-1}\sum_j 1/u_j.$$

Now substitute this expression for λ back into the FOC of the profit function and solve for x_i.

$$u_i x_i = \frac{S}{\lambda} - \frac{cS}{u_i \lambda^2}.$$

$$u_i x_i = \frac{S}{\frac{c}{N-1}\sum_j 1/u_j} - \frac{cS}{u_i\Big(\frac{c}{N-1}\sum_j 1/u_j\Big)^2}.$$

$$u_i x_i = \frac{S(N-1)}{c\sum_j 1/u_j} - \frac{S(N-1)^2}{u_i c\Big(\sum_j 1/u_j\Big)^2}.$$

$$x_i = \frac{S(N-1)}{u_i c\sum_j 1/u_j} - \frac{S(N-1)^2}{u_i^2 c\Big(\sum_j 1/u_j\Big)^2}.$$

$$x_i = \frac{S}{c}\frac{N-1}{u_i\sum_j(1/u_j)}\Bigg\{1 - \frac{N-1}{u_i\sum_j(1/u_j)}\Bigg\}.$$

This last equation provides a necessary and sufficient condition for good i to have positive output at equilibrium.

- Now for prices. since $p_i = \lambda u_i$, we can substitute in from our expression for λ. Thus,

$$p_i = \lambda u_i.$$

$$p_i = (\frac{c}{N-1}\sum_j 1/u_j)u_i.$$

Subtract c from both sides,

$$p_i - c = (\frac{c}{N-1}\sum_j 1/u_j)u_i - c.$$

$$p_i - c = c\Bigg[\frac{u_i}{N-1}\sum_j(1/u_j) - 1\Bigg].$$

- Back to the profit function,

$$S\pi_i = (p_i - c)x_i.$$

Substituting,

$$S\pi_i = \Bigg(\frac{cu_i}{N-1}\sum_j(1/u_j) - c\Bigg)\frac{S}{c}\frac{N-1}{u_i\sum_j(1/u_j)}\Bigg\{1 - \frac{N-1}{u_i\sum_j(1/u_j)}\Bigg\}.$$

$$S\pi_i = S\Bigg(\frac{u_i}{N-1}\sum_j(1/u_j) - 1\Bigg)\frac{N-1}{u_i\sum_j(1/u_j)}\Bigg\{1 - \frac{N-1}{u_i\sum_j(1/u_j)}\Bigg\}.$$

$$S\pi_i = S\Bigg\{1 - \frac{N-1}{u_i\sum_j(1/u_j)}\Bigg\}\Bigg\{1 - \frac{N-1}{u_i\sum_j(1/u_j)}\Bigg\}.$$

$$S\pi_i = S\left\{1 - \frac{N-1}{u_i \sum_j (1/u_j)}\right\}^2.$$

So this is the profit flow earned by firm i.

- So now consider a 2-period game where N firms produce positive output in period 1 such that the quality of all their goods is set to unity. No entry occurs before period 2 when firms compete and their level of sales in period 1 determines their quality level in period 2 such that

$$u_i = Max\{1, x^{1/\gamma}\}.$$

Where $\frac{1}{\gamma}$ represents the elasticity of perceived quality with respect to period 1 sales. Thus,

$$\frac{x_i}{u_i}\frac{du_i}{dx_i} = \frac{x_i}{x^{1/\gamma}}(1/\gamma)x^{1/\gamma - 1} = \frac{1}{\gamma}, \quad for\ x_i > 1.$$

Assume that all rival firms set quality level $\bar{u}$ and that firm i sets quality level u_i. Thus our profit function above becomes,

$$S\pi_i = S\left\{1 - \frac{N-1}{u_i \sum_j (1/u_j)}\right\}^2.$$

$$S\pi_i = S\left\{1 - \frac{N-1}{u_i((N-1)\frac{1}{\bar{u}} + \frac{1}{u_i})}\right\}^2,$$

because $N-1$ firms set quality $\bar{u}$ while firm i sets u_i. Thus,

$$S\pi_i = S\left\{1 - \frac{N-1}{(N-1)\frac{u_i}{\bar{u}} + 1}\right\}^2.$$

$$S\pi_i = S\left\{1 - \frac{1}{\frac{u_i}{\bar{u}} + \frac{1}{N-1}}\right\}^2.$$

$$S\pi_i = S\left\{1 - \frac{1}{\frac{1}{N-1} + \frac{u_i}{\bar{u}}}\right\}^2.$$

Where u_i is firm i's quality level and $\bar{u}$ is the rivals level of quality.

- Now if u_i also equaled $\bar{u}$, then this simplifies to:

$$S\pi_i = S\left\{1 - \frac{1}{\frac{1}{N-1} + 1}\right\}^2.$$

$$S\pi_i = S\left\{1 - \frac{1}{\frac{N}{N-1}}\right\}^2.$$

$$S\pi_i = S\left\{1 - \frac{N-1}{N}\right\}^2.$$

$$S\pi_i = S\left\{\frac{1}{N}\right\}^2 = \frac{S}{N^2}.$$

Thus,

$$\pi_i = \frac{1}{N^2}.$$

- Differentiating,

$$\frac{u_i}{\pi}\frac{d\pi}{du_i}\bigg|_{u_i=u_j=\bar{u}} = \bar{u}N^2[2(1/N^2)\frac{1/\bar{u}}{N^2/(N-1)^2}] = 2\frac{(N-1)^2}{N} = 2[N + \frac{1}{N} - 2].$$

[G-7.5] See graph in notes which shows that as S increases, we get convergence of the number of firms to some level $\tilde{N}$. This illustrates the relationship between the network effect parameter, γ, and equilibrium concentration. The figure shows the left and right sides of the following equation (derived in Technology and Market Structure, pg 384):

$$2[1 - \frac{1}{\gamma}(N + \frac{1}{N} - 2)] = \frac{N^2}{S}.$$

When $\gamma = \infty$, there are NO network effects and each firm earns profits of $\frac{S}{N^2}$ in each stage. For any finite γ, we obtain a nonconvergence result as follows. Denote $\tilde{N}(\gamma)$ as the unique root of the equation $N + \frac{1}{N} - 2 = \gamma$. Thus, N increases with S and approaches $\tilde{N}(\gamma)$ asymptotically as $S \longrightarrow \infty$. A lower bound to concentration is given by

$$\underline{C}_1 \geq \frac{1}{\tilde{N}(\gamma)}.$$

- Profit in a 2-stage game is therefore:

$$\Pi_i = (p - c)x_i + S\pi(u_i(x_i)|u_{-i}).$$

So total profit depends on profit this period, the first term, and profit next period, the second term. Note that the next period profit depends on the level of output in period 1. Thus by raising x_i today (lowering prices and profits), you enhance quality tomorrow and thus earn higher profits overall. This is analagous to the $F(u)$ we had before. It's the opportunity cost of profit foregone in period 1.

- In the basic cournot model with unit cost, c, in period 1, $c_i = 1$ and in period 2, $c_i = min(1, x_i^{-\theta})$. This is a simple isomorphism. Here θ replaces $\frac{1}{\gamma}$ and c_i replaces $\frac{1}{u_i}$. So Cournot model with quality is identical to the cournot model with unit cost.

8 Week 8: 4 Mar - 8 Mar

8.1 Modeling Market Structure

- The obvious features that we observe are 1) industries are differentiated in characteristics and this implies a different industrial structure. 2) Within an industry, the size distribution is usually skew.

- The literature starts with a paper by Gilbrat in 1931. He studied the size of business sales and plotted the distribution which he found to be heavily right skewed.

- Thus, via a method applied by Kapteyn, the Dutch Astronomer, he took the log of sales and plotted the probability distribution. The result was a Normal distribution. Thus the distribution of sales in an industry is said to be log-normal.

- We get the normal distribution via the central limit theorem. Gilbrat stated that the size of a firm x at time t is given by:

$$x_t = x_0 \cdot \underbrace{\theta_1 \cdot \theta_2 \cdot \theta_3 \ldots}_{iid\ proportional\ shocks} .$$

  Since the shocks are iid we get a normal distribution of x_t.

- Interpretation: If the percentage growth rate is uncorrelated with current size, then the size distribution will tend to a log-normal distribution. This is known as "Gilbrat's Law."

- Economists began to reject Gilbrat's law because they couldn't come up with a reason why growth should be positively or negatively correlated with firm size.

- The Literature in the 1970's. Gilbrat's Law was good as a rough approximation, ie no correlation between growth rate and size, BUT by the 1980's new data sets giving size and age of the firm suggested that:

  - 1) Big plants grow slower in percentage terms.
  - 2) Small plants have a high exit rate.
  - These two effects cancel each other out and thus we get no correlation between size and growth.

- Contemporaneous work on size distribution suggested that NO ONE SHAPE of distribution fits all industies well. (Schmalensce, Handbook of IO, 1991). Thus both theorists and empiricalists lost faith in Gilbrat's law.

- So we will use a bounds approach to get at Gilbrat's law. The agenda is as follows: Game Theory meets Independent Submarkets.

- We seek a SPNE in pure strategies if there are independence effects between submarkets. The simpliest example is a series of 4 islands each with a cement factory on it. The submarkets are completely independent. Suppose there are 4 firms viaing for a position on one of the islands, A, B, C, and D. In terms of structure we can get 4 possible NE outcomes:

$$\text{A,A,A,A} \Longrightarrow C_1 = 1.$$
$$\text{A,A,A,B} \Longrightarrow C_1 = \frac{3}{4}$$
$$\text{A,A,B,B} \Longrightarrow C_1 = \frac{1}{2}.$$
$$\text{A,B,C,D} \Longrightarrow C_1 = \frac{1}{4}.$$

- Note however that because the submarkets are independent and firms all identical, the first outcome is very unlikely as is the last outcome. The first requires that A gets "lucky" every time and wins the island and the fourth implies that A wins the first, but never wins again. Neither are statistically likely.
- We will find that in this set up, the lower bound to C_1 is much higher than $\frac{1}{4}$.
- We can proceed with two different approaches: Stochastic processes and a Game theoretic structure.

8.1.1 Stochastic Processes Approach

- How did the Growth of Firms tradition analyse this? It would allocate opportunities to firms with certain probabilities.
- Suppose with independent submarkets, we have a set up with firm A owning two plants and firm B owning one. A new plant is going to be built. Gilbrat's law says that $Prob(A) = 2Prob(B)$. Even without Gilbrat, any strategic disadvantage suffered by large firms should be unimportant. A large firm could always break up into smaller firms if there existed a disadvantage. (There are no diseconomies of scale). This is called the Replication Argument. At the very least we should be able to say: $Prob(A) \geq Prob(B)$.
- On the demand side, is size a disadvantage? The argument is that if A is large and cuts its price, it will not only steal B's customers, but also some customers from its other plants. This is a negative externality that we will have to pay attention to.
- With many independent submarkets in a large overall market, these demand side externalities can be avoided by the large firm not locating its plants in the close proximity to each other. Thus size is not a DISadvantage for a firm.

- We replace Gilbrat's law with the following: Our hypothesis is that the probability of capturing a new opportunity is non-decreasing in size. Consider an existing market with a small firm A and a large firm B already in the market. A new opportunity comes up and A and B compete with entrant firms C, D, and E for the opportunity. Gilbrat's law provided us with a rule for the competition between A and B, but we need a rule for the potential entrants as well. For this we follow Simon's benchmark.

- Simon et al says: The probability that a new entrant captures a new opportunity is constant over time. Call it P.

- Among active firms, large firms do not have a disadvantage. The bigger the advantage enjoyed by larger firms, the more unequal is the firm size distribution.

- We examine the results using the Lorenz Curve. Consider the case where all firms have the same probability (ie Absolute Growth Rate) then we get the least unequal distribution. The Lorenz Curve is plotted with the fraction of firms ranked from biggest to smallest on the horizontal axis and the fraction of plants owned on the vertical axis. If the line is a 45 degree line bisecting the graph, we have no inequality. But we will see that the curve derived is in the upper quadrant of the square which will not be derived since next week we will move to the Game Theoretic structure. The data clouds in this area as well and the mathematically derived curve serves as a bound to concentration. The cloud of points might shift for each country, but the edge is constant.

- As an example, let $k = 4$ and $C = 10$. The model solves out (which we will not need to know) and we get the following equation for the Lorenz Curve:

$$C_k \geq \frac{k}{N}(1 - ln(\frac{k}{N})).$$

  Where N is the total number of firms in the industry. Plugging in 4 and 10, we find that $C_4 \geq 0.77$. Thus the four firm concentration rate in this model must be at least 77 percent. Not the 40 percent that would occur if all firms shared the market equally. See excel spread sheet with different Lorenz curves for $N = 10, 15$, and 20. **[G-8.1]** **[Graph.]**

9 Week 9: 11 Mar - 15 Mar

9.1 A Game Theoretic Analysis of Firm Entry

- We consider a market with several independent/dependent submarkets. Some firms get large just by statistical accident. We would like to eliminate any asymmetries in the model. We will use the symmetry assumption to generate "Equal Treatment."
- The statistical model of last week led us to a lower bound to the Lorenz curve. That is, for any given size ranked firm, they had to own at least some fixed proportion of the plants in the industry.
- In terms of game theory, consider an island where there is enough room for exactly one firm to enter. We have two pure strategy equilibriums: I enters and II does not enter and vice versa. There is also another strategy in which NO ONE enters and the game starts again.
- Consider a dynamic entry game where firms face an entry cost function such that:

  $$F(t_0) = \epsilon e^{-rt_0}.$$

  Where ϵ represents the fixed cost of entry. The payoff to the firm is then:

  $$\Pi(t_0) = \int_{t_0}^{\infty} S(t)\pi e^{-rt} dt.$$

  Where π is the profit per capita and $S(t)$ is the population of the market at time t.
- We equate expected payoffs to costs to determine the optimal t_0, ie the time for entry to occur.
- To simplify the analysis, we look at the discrete case and then take the limit. Suppose entry can only occur at discrete steps in time, say $0, \Delta, 2\Delta$, etc.
- Solution: With N entrants, there are N pure strategy equilibria (one for each firm entering), and there is also a mixed strategy equilibrium which is best seen in the following game:
- Consider the Grab the Dollar Game. Two players have to decide to grab a dollar that is sitting on the table. If one grabs and the other doesn't then the grabber keeps the money. If neither grabs, they play again. If they both grab, they crash and each pays a penalty of a dollar. The crash case is the mixed strategy we are referring to.
- As $\Delta \longrightarrow 0$, each firm has probability $\longrightarrow \frac{1}{N}$ of entering so the event of a "crash" occuring where both firms enter the island has probability 0. Intuitively, this is because at time t_0, it is barely profitable for firm entry (it is the first point in time when profits become positive), so the cost of a crash is large so the crash will never occur at time t_0.

- Note that we have lost none of the pure strategy Nash equilibrium so far.
- To proceed with the analysis, we first make an assumption about the nature of the submarkets. We assume that all events that occur in the first submarket happen before the next submarket opens up. Thus you know the "size" of a firm at all time periods.
- If there are N firms already active in the market, assume the chance that any one of those firms is successful in the next available opportunity is $\frac{1}{N}$.
- See graph in notes. [**G-9.1**] Assume that every m^{th} opportunity is filled by a NEW Entrant. Thus for $m = 3$, entry of a new firm occurs at time $1, 4, 7$, etc. Equivalently we could say that the probability of a new entrant winning an opportunity is:

$$P = \frac{1}{m}.$$

- So consider the situation where submarket 4 opens up and a new entrant takes this opportunity. There are now 2 firms in the market. Thus the probability that either of them get the next two submarkets is $\frac{1}{2}$ each. Then at time 7, a new firm enters and again the new 2 submarkets will go to any one of the 3 active firms with probability $\frac{1}{3}$.
- We aim to calculate the expected size of a firm given its date of birth (entry into the market as a whole).
- So for a new entrant at time 4, we say that initially the firm has size 1. Then the expected size of this firm is as follows:

$$1 + \frac{1}{2}(1) + \frac{1}{2}(1) + \frac{1}{3}(1) + \frac{1}{3}(1) + \frac{1}{4}(1) + \frac{1}{4}(1) + \dots.$$

- This is just the sum of indicator variables. We use the rule that if $x, y \sim$ Poisson(λ_1 and λ_2), then

$$x + y \sim Poisson(\lambda_1 + \lambda_2).$$

  To use this theorem, we need for x and y to be "Approximately Independent." If you have a set of random variables, $x_1, x_2, x_3, x_4, \dots$, and then pick some function θ of these random variables such that θ depends on the x's in the following way: θ_2 depends on (x_1, x_2, x_3), θ_3 depends on (x_2, x_3, x_4), etc. THEN: if the x's are independent, the θ's are Approximately Independent.
- Define the size of the firm as:

$$X = 1 + r.$$

  Where 1 is the firm's entry size and r is a random variable representing how the firm's size grows over time. r is Poisson distributed or:

$$f_i(r) = \frac{e^{-\lambda_i}\lambda_i^r}{r!}.$$

Where i is an index showing the rank (age or date of entry) of a firm. And λ_i is defined as:

$$\lambda_i = \frac{1-P}{P}\left[\frac{1}{i} + \frac{1}{i+1} + \frac{1}{i+2} + \cdots + \frac{1}{N}\right].$$

Because $P = \frac{1}{m} \Rightarrow m = \frac{1}{P} \Rightarrow m-1 = \frac{1}{P} - 1 = \frac{1-P}{P}$. Where $m-1$ is the number of opportunities that open up each round for an already active firm to jump on. Thus with $m = 3$, the opportunities available to an active firm per round was 2. We multiply this number by the probability stream of an active firm winning the opportunity. Notice that as more and more firms enter, the probability that an active firm will attain the next opportunity is falling until we reach the final number of firms in the market, N, and the chance that active firms attain the last two opportunities is $\frac{1}{N}$.

- Recall that $f_i(r)$ is the size distribution of firm i. Thus the average size distribution of the industry is given by:

$$f(r) = \frac{1}{N}\sum_{i=1}^{N}\frac{e^{-\lambda_i}\lambda_i^r}{r!}.$$

- On supposed "standard calculation", substituting in for λ_i and simplifying, we have:

$$f(r) \longrightarrow P(1-P)^r.$$

And since $X = 1 + r$,

$$f(X) \longrightarrow P(1-P)^{X-1}.$$

And this is a geometric distribution which when we smooth it, becomes exponential. Thus we again have a size distribution BOUND as we had from the statistical analysis.

9.1.1 Testing the Interpretation

- Say we have submarkets with many firms in each. We can now define the entry game, and note that along the equilibrium path of the game, at various points (nodes) an entry event occurs.
- We denote a firm entering at points as the"first entrant", the "second entrant", etc as each firms "Role." Note that each role is filled by exactly one firm but not all roles are of equal size. Firm A could enter on Roles 1 and 2 and firm B could enter on Role 3. Firm A would be twice the size of B.

- To show that our analysis is valid, we would like to show that the submarkets are the key to forming this lower bound. If we look narrowly enough at an industry we see that firms lie along the 45 degree line of the Lorenz Curve but as we broaden our scope, we see the lower bound taking shape.

- We also have the "Externality Effect." If Toughness of Price Competition is LOW and the Degree of Substitution is HIGH, then ALL EQUILIBRIUM CONFIGURATIONS ARE OF THE FRAGMENTED FORM $(1, 1, 1, \ldots, 1)$. (Each firm is of minimum size). This is because the externalities are strong because building a second plant has strong negative externalities on the first existing plant.

- In the first part of the course, we developed a lower bound to concentration based on things like $R\&D$ intensity and product substitutability. In this section, we have introduced a new bound which adds to our old bound and restricts the number of equilibriums even more. See graph in notes. [**G-9.2**]

10 Week 10: 18 Mar - 22 Mar

10.1 Firm Exit

- For the last section of the course, we consider a declining industry and we study the way in which firms exit the industry.
- Graphically, imagine that the demand schedule is shifting in over time and collapses to zero at some point in the future.
- Consider N firms in an industry. We are intrerested in the order of exit. One idea would be to assume that firms exit in order of their profitability before the decline, with the least profitable leaving first. Another way of thinking, and the way we will model below, is that the largest firms are the ones most hurt by the decline in demand so they should be the first ones to leave.
- We begin with a (rather special) model by Ghemawat and Nalebuff. The setting corresponds to industries with homogeneous products. If we did not assume homogeneous products, the analysis would be complicated because the exit of one firm (or variety) might have positive externalities on substitute varieties. Also assume that each firm operates only one plant.
- Cost and Capacity will be the only difference between firms. The first question to ask: Is it possible to scale down a firm instead of exiting the industry? Usually in most industries, scaling down a plant is costly. In other words, cutting production in half does not necessarily cut costs in half. At least a large part of fixed outlays (overhead costs of running the plant) are a function of plant capacity rather than current output. Thus it may not be cost effective to scale down.

10.1.1 The Model

- The two key features of the model are 1) Single plant firms and 2) Plant capacity is related to overhead costs.
- Consider two firms in an industry of capacity K_1 and K_2 with $K_1 > K_2$. Thus firm 1 is the "Big firm" and firm 2 is the "Small firm."
- Let overhead cost be represented as a running cost of c per unit of capacity per unit of time. For simplicity assume constant marginal cost of zero.
- See graph in notes which shows supply and demand in the market and where the demand curve intersects the constant overhead cost curve c and yields zero profits for both firms. [**G-10.1**]
- As the demand schedule shifts in, prices fall eventually to c, then to marginal cost $= 0$.
- We now consider three scenerios. One in which both firms stay in the market, one where only the Big firm leaves and one where only the Small firm leaves.

- See graph in notes. **[G-10.2]** Define a time, t_0, as the time when price has just fallen to c so firms make zero profit.

- If both firms stay in the market, then at t_0, their profits will both be zero and then fall over time. Since firm 1 is the Big firm, his profits will fall faster since he is more effected by the decline in demand.

- If only firm 1, the Big firm, stays in the market, then since the other firm has left, this initially increases firm 1's profits up to some point. Beyond this point, his profits fall off and eventually go negative. If only firm 2, the Small firm, stays in the market, then since the other firm has left, this initially increases firm 2's profits up to some point below where firm 1's profits were if he was alone. Beyond this point, his profits fall off and eventually go negative. Note that because firm 1 is the Big firm, his profits fall off more quickly than firm 2 when they are alone.

- Define:

$$C_i(Z, t_0) \equiv \text{ Firm } i\text{'s cost from time } t_0 \text{ to } Z.$$
$$V_i(Z, t_0) \equiv \text{ Firm } i\text{'s profit from time } Z \text{ to when he exits.}$$

  Where Z is the time that the rival firm, firm j, exits.

- **[G-10.3]** So consider the two scenerios where both firms exist in the industry alone. Since the firm 1's profits fall off faster than firm 2's, he will reach zero profits faster than firm 2. The point at which this line crosses the axis is initially defined (from 2's perspective) as time Z, or the time of exit for firm 1. Thus, see graph, but the area in the triangle under firm 2's profit curve from time Z, call it t_1^*, to the time which his profit curves cross the zero barrier is $V_2(t_1^*, X_A)$. Time t_1^* is the latest possible time that firm 1 would exit the industry if he was in it alone.

- But now consider firm 2's position. He makes these positive profits $V_2(t_1^*, X_A)$ ONLY IF HE IS IN THE MARKET ALONE. Prior to t_1^*, firm 1 could make positive profits if he was in the market alone. However, he knows that firm 2 will stay in the market if the potential profits from being in the market alone outweigh the costs of being in it with firm 1. Thus firm 2 has the strategic advantage. We therefore equate firm 2's potential profits, V_2 with the cost of being in the market with firm 1 prior to time t_1^*. See graph for the relevant areas. Equating these areas,

$$V_2(t_1^*, X_A) = C_2(t_1^*, X_A),$$

  [G-10.4]

  we see that firm 1 will now decide that exit at time X_A is his dominant strategy. He knows that beyond this time, firm 2 will stay in the market because his expected profits, even beyond t_1^* (the latest possible time that firm 1 would exit the market) outweigh (exactly equal) the costs of staying in the market and making losses from time X_A to time t_1^*. Thus, we know that time X_A is now the new latest time that firm 1 will exit the market. From this point, we can show firm 2's expected profits and again work backwards by comparing those profits with losses from being in the market together. Thus firm 1 chooses to exit at time X_B. (And so on...)

- The backward induction argument: in each subgame described above, firm 1's optimal reply is to exit immediately. Thus repeating the argument backwards, and defining a sequence of exit times $(X_A, X_B, X_C, \dots)$, we find that in the SPNE, firm 1 exits at time t_0. ie, the Big firm exits the market as soon as price is driven down to cost c, the overhead cost.

10.1.2 Subsequent Literature

- In the later QJE article (1990), they extended this to multiplant firms. The new prediction is that the plants exit in order of size (the largest plants first). The Soda Ash industry study confirmed these results.

- In Lieberman, a study of the chemical industry found that the cost assumption (about scaling) is valid and this confirmed the G/N results.

- In Deily, he studied the steel industry and again found that the cost assumption was valid. However, here the pattern of exit suggested several determinants of exit but plant profitability seems to be the key. This results goes against G/N.

- In Baden-Fuller, they studied the industry for steel-casings. They found that the profitability factor was NOT a key element in the decision to exit (against Deily's findings). Rather, a key factor was the characteristics of the firm (as opposed to the plant). The most important characteristic was the level of diversification within the firm and how financially strong and stable was the firm. (These characteristics made the firms less likely to close plants).

- In Schary, she studied exit in the US cotton textile industry. Here the cost assumption does NOT apply since scaling down cotton production is not costly. She rejected a threshold profitability model, or in other words, the order by profitability hypothesis. She models a sequential decision process and considered several factors in turn and these internal firm decisions turned out to best predict exit.

- So in sum, exit purely based on firm profitability is too simple of a model. When the cost assumption of G/N applies, it seems that the size of the firm does play a large role in determining exit (though NOT in all cases). A general model of exit would have to be more complicated and involve size, profitability, and internal firm decisions.

10.2 Exam Details

- Choose 4 out of 8 questions.

- Technical material comes from the following articles and chapters: 1st Term: D'Aspremont, Green/Porter, Kreps/Wilson, and Axelrod. 2nd Term: TMS 2,3,11; Ghemawatt/Nalebuff.